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Theoretical Model of the Electric Field of Stray Currents in Underground Installations in Urban Environments

S. Bjelić[1], B. Jovanović[2], Z. Bogićević[3*], N. Marković[2] and I. Bjelić[4]

[1]Faculty of Technical Sciences, University of Priština, Kneza Miloša 7, 38220 Kosovka Mitrovica, Serbia.
[2]High Technical School of Professional Studies, Urosevac, 38218 Leposavic, Serbia.
[3]High Technical School of Professional Studies, Nušićeva 6, 38227 Zvecan, Serbia.
[4]Faculty of Civil Engineering and Architecture, University of Nis, Aleksandra Medvedeva 14, 18000 Nis, Serbia.

Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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ABSTRACT

Aims: Paper presents the graphically observe the impact of parameters on equipotential forces. By adjusting the corresponding parameters their operation in the space can be significantly reduced. Graphics of positive and negative features equalizing line is obtained using Wolphram alpha computer program and application support functions.
Methodology: The intensity of corrosion and destruction of metal pipes is determined by the difference between the potential of soil and pipes (pipe voltage). The structure and various forms of underground pipe installations make it difficult to solve this task analytically, so that it becomes virtually impossible. Therefore, the first step in solving the task is defining a theoretical model for

**Corresponding author: E-mail: zocabogi@gmail.com;*

determining the value of electrical potential on the elements of underground installations. Further proceedings can determine the order and the minimum number of elements of cathode protection by which the voltage at the isolation layer of pipe is limited to allowed values.

Conclusion: To ensure the potential change along the axis of the tube, the metal surface should be divided into smaller elements of the same length, and to each element of length Δl the longitudinal resistance and conductivity should be added.

Keywords: Electric field; metal pipes; underground installation; parameters; longitudinal resistance; longitudinal conductivity.

1. INTRODUCTION

Spatial and temporal distribution of thermal energy, electromagnetic energy, propagation of light, sound and physical processes in buildings and technical systems are described by differential equations. Out of elliptic equations can be obtained solutions for stationary processes (no change in time) of a parabolic equation and hyperbolically for non-stationary processes. In electromagnetic fields, Maxwell's equations determine the processes and the size [1-4]: E is electric field strength, D is electrostatic induction and φ is electric scalar potential. Computers provide good support for analysis of the distribution of electromagnetic quantities and parameters in space and time, and the Laplace equation is used for solving tasks. Knowing the distribution of electric fields that arise from stray currents in the soil is necessary in design of grounding or corrosion protection (cathodes protection) [5,6].

2. DISPLAY OF ELECTRIC FIELDS OF STRAY CURRENTS

For aquantity (u), $\Delta u = 0$ is a simple elliptic equation with an operator: $\Delta = \nabla^2$ (Lapl. operator) which can be applied to both scalar and vector functions. In the system of Cartesian coordinates (x, y, z) for scalar function $\varphi(x, y, z)$ Laplace equation is [7,8]:

$$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = 0 . \qquad (1)$$

In the cylindrical system (r, α, z) coordinates, Laplace equation has the following form:

$$\nabla^2\varphi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) + \frac{1}{R^2}\frac{\partial^2\varphi}{\partial\alpha^2} + \frac{\partial^2\varphi}{\partial z^2} = 0 . \qquad (2)$$

where $\varphi = \varphi(r, \alpha, z)$ is a function of potential.

One of the elliptic equations is Poisson equation which for linear isotropic matrices of the magnetic characteristics $(\mu_x = \mu_y = \mu_z = \mu = const.)$ has the following form:

$$\nabla^2 A = -\mu_a J . \qquad (3)$$

where A is magnetic vector potential, J is vector of the current density, $\mu_a = \mu \cdot \mu_0$ absolute magnetic permeability of the matrix.

Fornon-linear matrices, $\nabla^2 A = -\mu_a J$, Maxwell equations are as follows:

$$rot\frac{1}{\mu}rot\vec{A} = \mu_0\vec{J} \text{ or}$$

$$-\frac{1}{\mu}\nabla^2\vec{A} + grad\left(\frac{1}{\mu}\right) \times rot\vec{A} = \mu_0\vec{J} . \qquad (4)$$

Magnetic vector-potential A is a quantity vector and in the Cartesian coordinate system $\vec{A} = iA_x + jA_y + kA_z$, and vector of the current density is as follows $\vec{J} = iJ_x + jJ_y + kJ_z$. Poisson equation is divided into three equations with scalar quantities (A_x, A_y, A_z). If we suppose that in the model of electric element vector of the current density and magnetic vector potential contain only z -component solving the task is reduced to solution on the level. Poisson equation the Cartesian system is as follows:

$$\frac{1}{\mu}\left(\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2}\right) = -\mu_0 J_z . \qquad (5)$$

Laplace and Poisson equations in the model of power devices have a unique solution if they are complemented with the following boundary conditions:

1. The boundary conditions of the first order (Dirichlet's) – on the border is G set value of the required function, that is $\varphi = f_1(x, y, z)$ and points with Cartesian coordinates (x, y, z) belong to the border G. The condition $\varphi \;= 0$ represents the condition of homogeneity.
2. The boundary conditions of the second order (Neumann's) – on the border G are set changes of the required function through the perpendicular on the border $\frac{\partial\varphi}{\partial n} = f_2(x, y, z)$ and points with coordinates (x, y, z) belong to the border G. Here the condition of homogeneity is as follows $\frac{\partial\varphi}{\partial n} = 0$.
3. The boundary condition of the third order $\frac{\partial\varphi}{\partial n} + f_3(\varphi) = f_4(x, y, z)$ where points with coordinates (x, y, z) belong to the border G where as on the border of the domain in the model the conditions containing border conditions – of the first, second and third order can be set. In environments with linear EM characteristics parameters R, L, C, G, M are constant, ie. passive. If the characteristics of the elements and the environment depend on the time, parameters of resistance $R = R(t)$, inductance $L = L(t)$ and capacitance $C = C(t)$, conductivity $G = G(t)$ mutual inductance $M = M(t)$ of differential equations, are time functions. Processes in nonlinear circuits elements are defined by the following functions: $R = R(i)$, $R = R(v)$, $R = R(i)$, $G = G(v)$, $M = M(i)$. When the parameters are linear, linear differential equation of the n-order for description of the transient state of the circuit is:

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + ... + a_1 x^2 + a_0 x = f(t). \quad (6)$$

where $x = x(t)$ is the response, $f(t)$ is excitation and $a_n, a_{n-1}...., a_1, a_0$ constants.

The elements have complex geometric forms of external and internal border areas and a large number of sub-areas of different electrostatic, magnetic and conductive characteristics. Spatial distribution of electrical quantities in devices is determined using the finite element method. The finite element method is used to determine the initial forms and criteria of calculating quotients and then to test solutions of differential equations and to determine the "projections" or approximate solutions in finite space.

Discrete model of continuous area is formed as follows:

1. In the area defined by the model a finite number of points is fixed. These points are nodes of computer network, covering the area of modeling.
2. The size of the continuous variable value in the node is defined.
3. The area in which modeling of quantities takes place is divided into elements, "finaly elements", and the final number of sub-areas is obtained. Finite Element Method, FEM, which is based on approximation of continuous functions (potential, temperature etc.), defines a discrete model with a set of partial-continuous functions, which are determined for a finite number of sub-areas. Polynomials are used as a function of the elements since the classification of finite elements is implemented according to the order of the polynomials.

There are three groups of elements: simplex-free-simple, complex-complex, multiplex-multiple elements.

Classic two-dimensional simplex-element is a right-angled triangle with three nodes. For arbitrarily selected j-th order node numbering of the nodes is counter-clockwise. The values in the nodes are scalar quantities φ marked with (Φ_j, Φ_j, Φ_k) and the coordinates of the three nodes are as follows: (X_i, Y_i), (X_j, Y_j), (X_k, Y_k) which enables determination of form functions using node coordinates of the selected computer network.

The characteristic feature of electrical and gas installations is a network of metal pipes deployed in half-space $(x<0)$, parallel to the surface $(x=0)$ and distance (m) from the surface of the soil. Electric potential $\varphi=\varphi(x,y,z)$ in the half-space underneath the soil surface formed by the stray currents on the pipes or their environment, is determined by the values of the longitudinal specific electrical conductivity:

$\sigma=\sigma_1 - in\ \ metal\ \ cylinder\ \ of\ \ thepipe,$

$\sigma=\sigma_2 - in\ \ conductive\ \ metal\ \ parts\ on\ surface\ earth,$

$\sigma=\sigma_3 - in\ \ isolation\ \ sheath, around\ \ the\ \ tube,$

$\sigma=\sigma_4 - in\ \ the\ \ soil, around\ \ the\ \ tube,$

The value of the potential in an arbitrary environment satisfies the equation:

$$\nabla^2\varphi=\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0/\cdot\sigma.\Leftrightarrow$$
$$\nabla^2(\sigma\cdot\varphi)=\frac{\partial}{\partial x}\left(\sigma\frac{\partial\varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\sigma\frac{\partial\varphi}{\partial y}\right)+\frac{\partial}{\partial z}\left(\sigma\frac{\partial\varphi}{\partial z}\right)=0 \quad (7)$$

This value of the potential satisfies the equation at all points in space, except in the point which connects the power source with a metal pipe (for cathodic protection) and mobile contact points of trolleybus on trolley line in electric transport (during the movement of electric locomotives) in which the potential has its own peculiarities. On the ground surface, for $(x=0)$ is $(\partial\varphi/\partial n)=0$, i.e. at all points except at points where the power source is grounded (e.g. cathodic protection). Electrostatic copying method (mirror image), static field can be replaced by a field formed by two parallel pipes of the same potential in a homogeneous environment (the ground) if the electrical source is infinitely distant.

Complex potential of the electrostatic field is $\breve{W}(\Omega)\Leftrightarrow\hat{W}(x,y,z)\Leftrightarrow\varphi(x,y,z)$ [1,2,5]:

$$W(\Omega)=\frac{I_o}{2\pi\sigma}\ln\frac{\theta_1\left(i\frac{\Omega-\Omega_0}{2h},i\frac{a}{h}\right)}{\theta_2\left(i\frac{\Omega-\Omega_0}{2h},i\frac{a}{h}\right)}\cdot \quad (8)$$

$$\Omega(z)=\frac{-ih}{\ln\frac{m-\sqrt{m^2-R^2}}{m+\sqrt{m^2-R^2}}}\ln\frac{z-\sqrt{m^2-R^2}}{z+\sqrt{m^2-R^2}}+i\frac{h}{2}=u+i\vartheta \quad . \quad (9)$$

$\Omega(z)$ is a function of copied plane at infinity, to the infinite line with h width, for the exclusion of circular cross-section tubes and pipes transferring a higher contour of the tube to a higher limit of the line $\vartheta=h$ and lower contour of the tube-to the lower limit of the line $\vartheta=0$. During this, the infinitely distant point is transferred on the line as a series of equally distant points with coordinates $\Omega_k=j\frac{h}{2}+2ak$ (j is imaginary unit, k is arbitrary integer). Fig. 1 presents determination of potential value of long straight tube with radius $R[m]$, covered by a thin layer of insulation and placed at the depth of $m=h$ from the surface of the ground where $2y_1$ is coordinates of the tube surface, $g[Sm]$ is parameter of transverse conductivity as the equivalent of the insulating layer of pipe and separated layers of ground on the length of the pipe element Δl , $r[\Omega/m]$ is longitudinal parameter of resistance of the tube element Δl [9-11].

The voltage on the surface of the pipe is a difference between the potential of pipe φ_0 and the reference potential of the ground φ_z ; $V_0(z)=\varphi_0-\varphi_z$; $V_0\Leftrightarrow\varphi_0$; $\varphi_z\approx 0$. The voltage on the surface of a layer of soil is a difference between potential of the pipe and the ground potential $V_1(z)=\varphi_1-\varphi_z$; $V_1\Leftrightarrow\varphi_1$; $\varphi_z\approx 0$. Fig. 1 shows: $V_0+\frac{\partial V_0}{\partial z}dz$ voltage and its growth by axis (z), $i+\frac{\partial i}{\partial z}dz$ current that flows from the ground to the pipe by axis (z) and its growth.

Solving the task is easier if we introduce the replacements:

$$a=\frac{2\pi h}{\ln\frac{m+\sqrt{m^2-R^2}}{m-\sqrt{m^2-R^2}}},b=\sqrt{m^2-R^2}$$
$$\Omega_0=j\frac{h}{2},\overline{\Omega}_0=-j\frac{h}{2} \quad . \quad (10)$$

and when the function θ_1 is presented in the form of a series with members of the order:

$$\theta_1(\xi_i)= \\ =2\left[q^{\frac{1}{4}}\sin\pi\xi_i - q^{\frac{9}{4}}\sin 3\pi\xi_i + q^{\frac{25}{4}}\sin 5\pi\xi_i - ...\right] \quad (11)$$

$$q=e^{-\pi\frac{a}{h}}=e^{-\frac{2\pi^2}{\ln\frac{m+\sqrt{m^2-R^2}}{m-\sqrt{m^2-R^2}}}}=e^{-\frac{2\pi^2}{\ln\frac{m+b}{m-b}}}. \quad (12)$$

$$\xi_1=i\frac{\Omega-\Omega_0}{2h}=\frac{\ln\frac{z-\sqrt{m^2-R^2}}{z+\sqrt{m^2-R^2}}}{2\ln\frac{m-\sqrt{m^2-R^2}}{m+\sqrt{m^2-R^2}}}=\frac{\ln\frac{z-b}{z+b}}{2\ln\frac{m-b}{m+b}} \quad (13)$$

$$\xi_2=i\frac{\Omega-\overline{\Omega}_0}{2h}=\frac{\ln\frac{z-\sqrt{m^2-R^2}}{z+\sqrt{m^2-R^2}}}{2\ln\frac{m-\sqrt{m^2-R^2}}{m+\sqrt{m^2-R^2}}}-\frac{1}{2}= \quad . \quad (14)$$

$$=\frac{\ln\frac{z-b}{z+b}}{2\ln\frac{m-b}{m+b}}-\frac{1}{2}=\xi_1-\frac{1}{2}$$

$$\theta_1(\xi_2)=\theta_1\left(\xi-\frac{1}{2}\right)= \\ =-2\left[q^{\frac{1}{4}}\cos\pi\xi_i - q^{\frac{9}{4}}\cos 3\pi\xi_i + q^{\frac{25}{4}}\cos 5\pi\xi_i + ...1\right]= \quad (15) \\ =-\theta_2(\xi_1)$$

By changing these values $q,\xi_1,\xi_2,\theta_1,\theta_2$ the term (8), the function of transfer from domain (Ω) to domain (z) is obtained, i.e. according to correlation formula $W(\Omega)\Leftrightarrow W(z)$:

$$W(z)=\frac{I_o}{2\pi\sigma}\ln\frac{\theta_1\left(\xi_1,i\frac{a}{h}\right)}{\theta_2\left(\xi_2,i\frac{a}{h}\right)}+i\frac{I_0}{2\sigma}=\frac{I_0}{2\pi\sigma}\times \\ \times\left[\ln\frac{\sin\pi\xi_1-q^2\sin 3\pi\xi_1+q^6\sin 5\pi\xi_1 \_ q^{12}\sin 7\pi\xi_1+...}{\cos\pi\xi_1+q^2\cos 3\pi\xi_1+q^6\cos 5\pi\xi_1+q^{12}\cos 7\pi\xi_1+...}+i\pi\right] \\ W(z)=\frac{I_0}{2\pi\sigma}\ln\frac{\sum_{n=1}^{\infty}(-1)^n q^{\frac{(2n-1)^2-1}{4}}\sin(2n-1)\pi\xi_1}{\sum_{n=1}^{\infty}q^{\frac{(2n-1)^2-1}{4}}\cos(2n-1)\pi\xi_1}$$

(16)

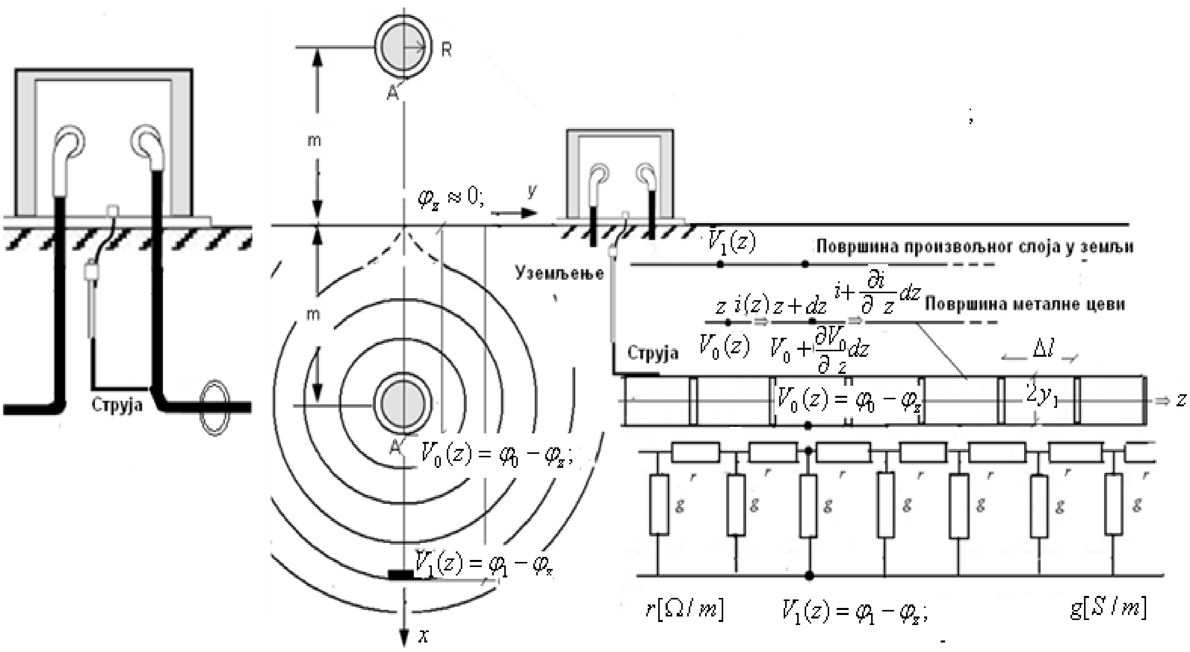


Fig. 1. Basic model for determining impact of stray currents on the formation potential on metal pipes in underground installations

If the values m and R are real for example ($m = 4R$, $R = 0{,}5m$) the quantity (q) is determined:

$$q = e^{-\pi\frac{a}{h}} = e^{-\frac{2\pi^2}{\ln\frac{m+\sqrt{m^2-R^2}}{m-\sqrt{m^2-R^2}}}} = e^{-\frac{2\pi^2}{\ln\frac{m+b}{m-b}}} = e^{\frac{-2\pi^2}{\ln\frac{4+\sqrt{15}}{4-\sqrt{15}}}} = \quad (17)$$

$$= e^{-4.77824} \approx 8.41 \cdot 10^{-3} = \frac{1}{118{,}895}$$

$$q^2 = 7.0728 \cdot 10^{-5} \approx 10^{-4}. \quad (18)$$

In the expression $W(z)$ all members under the sign of the logarithm in the numerator and denominator except for the first one scan be ignored as this does not affect accuracy of the calculation:

$$W(z) = \frac{I_0}{2\pi\sigma}\left(\ln tg\,\pi\xi_1 + i\pi\right). \quad (19)$$

$$\pi\xi_1 = \frac{a}{4h}\ln\frac{z+b}{z-b} =$$

$$= \frac{a}{4h}\left[\ln\sqrt{\frac{(x+b)^2+y^2}{(x-b)^2+y^2}} - i \cdot arctg\frac{2by}{x^2+y^2+b^2}\right] = c - id \quad (20)$$

$$c = \frac{a}{8h}\ln\frac{(x+b)^2+y^2}{(x-b)^2+y^2}$$

$$d = \frac{a}{4h}arctg\frac{2by}{x^2+y^2+b^2} \quad (21)$$

$$tg\,\pi\xi_1 = tg(c-id) = \frac{\sin 2c - i sh2d}{\cos 2c + ch2d} = \sqrt{\frac{ch2d - \cos 2c}{ch2d + \cos 2c}}\, e^{-i arctg\frac{ch2d}{\sin 2c}} \quad (22)$$

By changing the last expression into (19) the following is obtained:

$$W(z) = \frac{I_0}{2\pi\sigma} \times$$

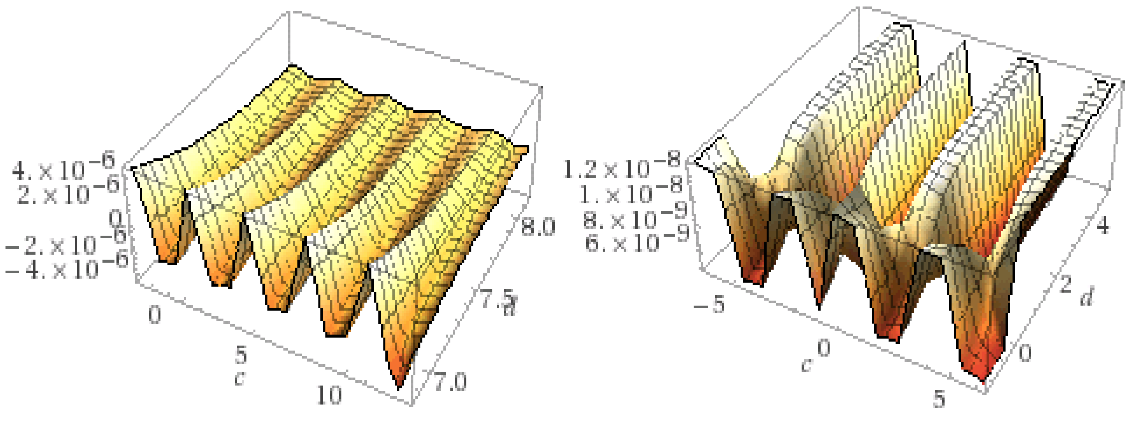
$$\times\left[\ln\sqrt{\frac{ch2d - \cos 2c}{ch2d + \cos 2c}} + i \cdot \left(\pi - arctg\frac{sh2d}{\sin 2c}\right) + K_0\right] =$$

$$= \varphi(x,y) + i\,\psi(x,y) \quad (23)$$

where

$$\varphi(x,y) = \frac{I_0}{4\pi\sigma}\ln\frac{ch2d - \cos 2c}{ch2d + \cos 2c} + K_0$$

$$\psi(x,y) = \frac{I_0}{2\pi\sigma}\left(\pi - arctg\frac{sh2d}{\sin 2c}\right) \quad (24)$$



a) b)

Fig. 2. For $\varphi^* = 1$, a) real $\varphi(x,y) = \frac{I_0}{4\pi\sigma}\ln\frac{ch2d - \cos 2c}{ch2d + \cos 2c} + K_0$ is presented according to (24) and b) imaginary part $\psi(x,y) = \frac{I_0}{2\pi\sigma}\left(\pi - arctg\frac{sh2d}{\sin 2c}\right)$ according to (24), this function for $I_0 = 1(A)$

Assuming that potential at point (0,0) is equal to zero $\varphi(0,0)=0$ is:

$$d(0,0)=\frac{a}{4h}arctg\frac{0}{-b^2}=\frac{a\pi}{4h},c(0,0)=\frac{a}{8h}\ln 1=0$$
$$\sin 2c=0,\varphi(0,0)=\frac{I_0}{4\pi\sigma}\ln\frac{ch\frac{a\pi}{2h}-1}{ch\frac{a\pi}{2h}+1}+K_0=0 \quad . \tag{25}$$
$$K_0=\frac{I_0}{4\pi\sigma}\ln\frac{ch\frac{a\pi}{2h}+1}{ch\frac{a\pi}{2h}-1},W(z)=\varphi(x,y)$$

According to (23) real value of potential is $W(z)=\varphi(x,y)$. φ-value of that area at the level of cross-section of the pipe is as follows:

$$\varphi(x,y)=\mathrm{Re}\{W(z)\}$$
$$\varphi(x,y)=\frac{I_0}{4\pi\sigma}\ln\frac{(ch2d-\cos 2c)\left(ch\frac{a\pi}{2h}+1\right)}{(ch2d+\cos 2c)\left(ch\frac{a\pi}{2h}-1\right)} \tag{26}$$

where

$$a=\frac{2\pi h}{\ln\frac{m+\sqrt{m^2-R^2}}{m-\sqrt{m^2-R^2}}}=\frac{2\pi h}{\ln\frac{m+b}{m-b}}\,. \tag{27}$$
$$b=\sqrt{m^2-R^2}$$

2.1 Deriving an Expression for Equipotential Lines Including a Metal Pipe

At greater distances from the tube, form of the equipotential line is a circle but near the pipe are lines of another form. If $z>>R$, $z>>m$ (19) the following is obtained:

$$\ln tg\pi\xi_1=\ln\left(tg\frac{a}{4h}\ln\frac{z+b}{z-b}\right)=\ln\left(tg\frac{a}{4h}\ln\frac{1+b/z}{1-b/z}\right)\approx$$
$$\approx\ln tg\frac{ab}{2hz}\approx\ln\frac{ab}{2h}-\ln z=\ln\frac{ab}{2h}-\ln r-i\upsilon \quad . \tag{28}$$

where

$$r=\sqrt{x^2+y^2}\ ,\ \upsilon=arctg\frac{y}{x}\,. \tag{29}$$

When $z>>R$, $z>>m$ equation of equipotential lines can be obtained if the tube is replaced by two parallel statically loaded axes with the electric charge $\pm\frac{\tau}{2}$ at mutual distance $2n$:

$$\varphi_1(x,y)=$$
$$=-\frac{\tau}{4\pi\sigma}\ln\sqrt{(x^2+y^2+n^2-2nx)(x^2+y^2+n^2+2nx)}+K_1 \quad . \tag{30}$$

Assuming that the surface of the ground $\varphi_1(0,0)=0$, the following is obtained:

$$\varphi_1(0,0)=-\frac{\tau}{4\pi\sigma}\ln n^2+K_1=0\,,\ K_1=\frac{\tau}{8\pi\sigma}\ln n^4\,. \tag{31}$$

Therefore, the general solution is:

$$\varphi_1(x,y)=$$
$$=\frac{\tau}{8\pi\sigma}\ln\frac{n^4}{(x^2+y^2+n^2-2nx)(x^2+y^2+n^2+2nx)} \tag{32}$$

Relation of the equipotential line, which passes the point $(0,y_1)$ is obtained through calculating the potential at that point $\varphi_1(0,y_1)$, (formula (26)):

$$\varphi_1=\varphi(0,y_1)=$$
$$=\frac{\tau}{4\pi\sigma}\ln\frac{\left(ch\frac{a}{2h}arctg\frac{2by_1}{y_1^2-b^2}-1\right)\left(ch\frac{a\pi}{2h}+1\right)}{\left(ch\frac{a}{2h}arctg\frac{2by_1}{y_1^2-b^2}+1)\right)\left(2ch\frac{a\pi}{2h}-1\right)} \tag{33}$$

So we obtain:

$$\varphi_1(x,y)=$$
$$=\frac{\tau}{8\pi\sigma}\ln\frac{n^4}{(x^2+y^2+n^2-2nx)(x^2+y^2+n^2+2nx)} \quad . \tag{34}$$

If the calculation is done in the unit system assuming that the value is $\frac{\tau}{4\pi\sigma} = \varphi_I$, is obtained by:

$$\varphi_{\bullet 1}(x,y) = \frac{\varphi_1(x,y)}{\varphi_I} = \\ = \frac{1}{2}\ln\frac{n^4}{\left(x^2+y^2+n^2-2nx\right)\left(x^2+y^2+n^2+2nx\right)} \quad (35)$$

$$\frac{n^4}{\left(x^2+y^2+n^2-2nx\right)\left(x^2+y^2+n^2+2nx\right)} = \\ = e^{2\varphi_{\bullet 1}(x,y)} \quad (36)$$

Then, after solving:

$$\left(x^2+y^2+n^2-2nx\right)\left(x^2+y^2+n^2+2nx\right) = \\ = n^4 e^{-2\varphi_{\bullet 1}(x,y)} = C_1 \quad (37)$$

By introducing shifts $x = \nu n$, $y^2 = \eta$ is obtained by:

$$\left(\nu^2 n^2 + n^2 - 2n^2\nu + \eta\right)\left(\nu^2 n^2 + n^2 + 2n^2\nu + \eta\right) = C_1$$

$$\eta^2 + 2n^2\left(\nu^2+1\right)^2 + n^4\left(\nu^2-1\right)^2 - C_1 = 0.$$

$$\eta = \sqrt{n^4\left(\nu^2+1\right)^2 - n^4\left(\nu^2-1\right)^2 + C_1} - n^2\left(\nu^2+1\right) = \\ = \sqrt{4\nu^2 n^4 + C_1} - n^2\left(\nu^2+1\right) \quad (38)$$

$$y = \sqrt{\eta} = \\ = \pm\sqrt{\sqrt{4\nu^2 n^2 + n^4 e^{-2\varphi_{\bullet 1}(x,y)}} - n^2\left(\nu^2+1\right)} = \\ = \pm n\sqrt{\sqrt{4\nu^2 + e^{-2\varphi_{\bullet 1}(x,y)}} - \nu^2 - 1} \quad (39)$$

By introducing the values $x = 0$, $\nu = 0$:

$$y_1 = n\sqrt{e^{-\varphi_{\bullet 1}(x,y)} - 1} \\ n = \frac{y_1}{\sqrt{e^{-\varphi_{\bullet 1}(x,y)} - 1}} \\ \nu = \frac{x}{n} = \frac{x\sqrt{e^{-\varphi_{\bullet 1}(x,y)} - 1}}{y_1} \quad (40)$$

If these values are changed into the equation of equipotential lines (39) the solution is obtained which determines equipotential lines, which pass through point $(0, y_1)$ whose potential is $\varphi(0, y_1) = \varphi_1$ when $y_1 >> R$:

$$y = \pm\frac{y_1}{\sqrt{e^{-\varphi_{\bullet 1}(x,y)} - 1}} \times \\ \times\sqrt{\sqrt{\frac{4x^2\left(e^{-\varphi_{\bullet 1}(x,y)} - 1\right)}{y_1^2} + e^{-2\varphi_{\bullet 1}(x,y)}} - \frac{x^2\left(e^{-\varphi_{\bullet 1}(x,y)} - 1\right)}{y_1^2} - 1} \quad (41)$$

By drawing the graphics of equipotential lines, according to the expression (41), for $y_1 \geq 20R$, it can be established that these lines become semicircles with centre on the surface of the ground, Fig. 1. Graphics of positive and negative function, equipotential line can be obtained using Wolphram alpha computer program for the assumed $\varphi_{\bullet 1} = 1$ and value $x = -0{,}5$; 0,0; 0,5 and $y_1 = -1$; 0,0; 1 and application of auxiliary function:

$$y = \pm\frac{y_1}{\sqrt{e^{-1} - 1}}\sqrt{\sqrt{\frac{4x^2\left(e^{-1}-1\right)}{y_1^2} + e^{-2}} - \frac{x^2\left(e^{-1}-1\right)}{y_1^2} - 1} = \\ = \frac{y}{\sqrt{\frac{1}{e} - 1}}\sqrt{\sqrt{\frac{4x^2\left(\frac{1}{e}-1\right)}{y^2} + \frac{1}{e^2}} - \frac{x^2\left(\frac{1}{e}-1\right)}{y^2} - 1} \quad (42)$$

For values $x = -0{,}5$, 0, 0,5 and $y = -1$; 0; 1, we obtain a graphic overview of function as shown in the Fig. 4.

If we take into account the assumptions adopted in the derivation of the expression (39), one can show that in reality potential changes a little along the pipe if the grounded tape is distant enough from the tube. Map field changes in the vicinity of points of divarication routes of metal pipes but even though it does not depend on the position of distant grounded poles, the potential will be different in the same points where different cross-sections of tubes merge: the shape of the field in the vicinity of the cylindrical tube is little influenced by distant sources of electric current. The electric field can be divided into two parts:

1) The field that includes the tube, insulation and a close layer of soil, limited by the surface area that are formed by equipotential lines equally distant from the axis of the tube;
2) Field in the rest of the ground.

For each of these parts of the field there should be created a special model, but both models have to be merged in order to obtain the solution of tasks simultaneously. Simulating can determine the voltage to which the insulating part of the tube is exposed, or the potential difference between two equipotential lines in each cross section of tube.

The distribution of electric fields strength between the equipotential lines is not interesting because of the unchanging form of the field in the vicinity of the tube; field can be considered two-dimensional with coordinates that are measured along the axis of the tube and along an arbitrary current line.

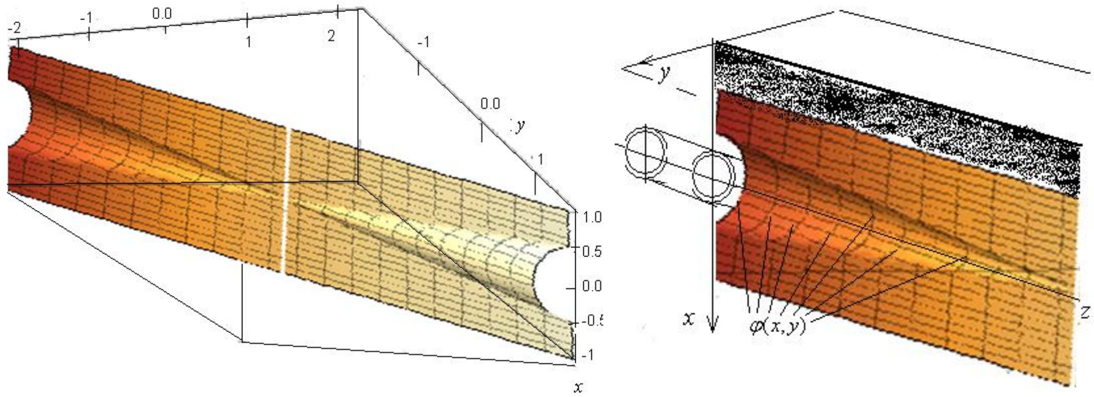


Fig. 3. Overview of elementary part of equipotential lines $\varphi^* = 1$

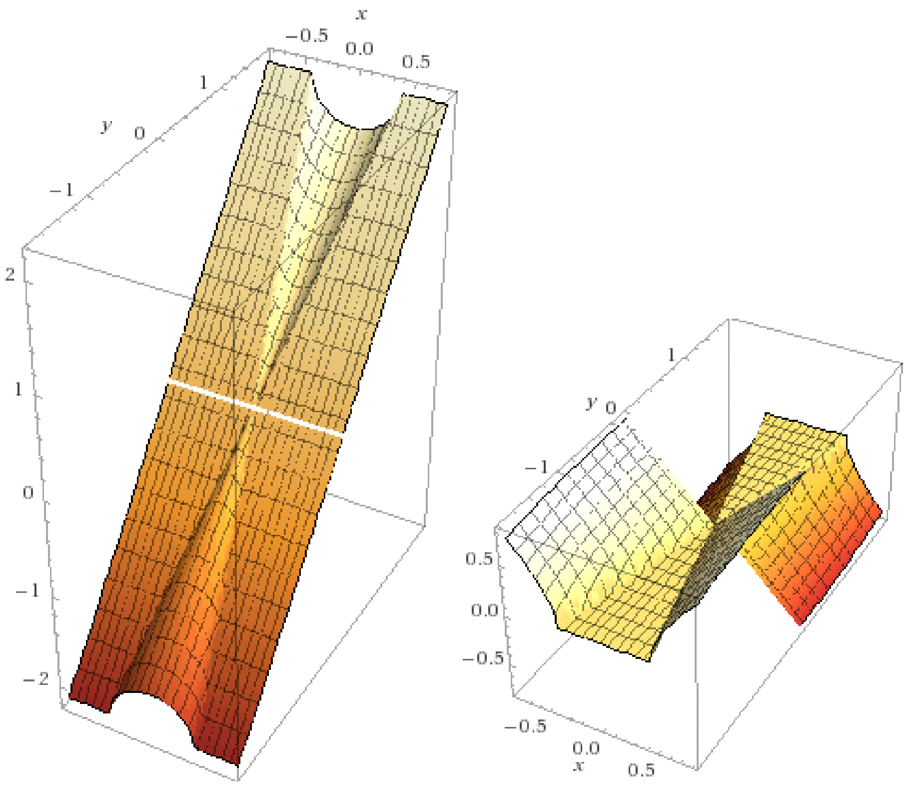


Fig. 4. Real and imaginary part of this function for $\varphi^* = 1$

The value of the voltage on the surface of the tube $V_0 \Leftrightarrow \varphi_0$ and the voltage on the surface of separated layers of ground $V_1 \Leftrightarrow \varphi_1$; can be determined from Fig. 1. For the tube element Δl the following applies:

$$V_0 - \left(V_0 + \frac{\partial V_0}{\partial z} dz \right) = r \cdot i \cdot dz \Leftrightarrow -\frac{dV_0}{dz} = r \cdot i \quad (43)$$

$$i - \left(i + \frac{\partial i}{\partial z} dz \right) = g(V_0 - V_1) \cdot dz \Leftrightarrow -\frac{di}{dz} = rg(V_0 - V_1) \quad (44)$$

After differentiation and appropriate replacement:

$$\frac{d^2 V_0}{dz^2} = rg(V_0 - V_1). \quad (45)$$

Equation (45) is similar to the equation of electricity transmission through long distance electricity transmission lines and potentials defined by it can be modeled using the voltage distribution in a simple circuit containing the longitudinal resistance (r) and transverse conductivity (g)[12,13].

In the rest of the ground field is three-dimensional, and an expression for the potential corresponds with the Laplace equation. This field can be modeled as a field of current using simulation with the aid of computers. The simulation model should include two main parts [14,15]:

1) A simple serial connection of partial pipe elements;
2) The parameters of elements of the remaining part, interconnected by the requirement to maintain the same potential at the connecting surfaces.

Potential or voltage $V_1(x, y)$ on that arbitrary surface layer of ground changes only along the axis of the tube. To change the potentials along the axis of the tube, the metal surface should be divided into smaller elements of the same length Δl, and each element has the same longitudinal resistance and conductivity.

3. RESULTS AND DISCUSSION

Although the process of flowing away of stray currents through the metal tube has a dynamic character, approximate solution is obtained by means of an electrostatic model for determining the voltage on the insulating layer of metal pipes in underground installations. Comparing multiple types of order of protective devices, we can determine the optimal order. Lengths of metal pipes in underground installations are tens and hundreds of kilometers long, and pipe sizes have a finite size: diameter of less than 1 m, thickness of the insulating sheath does not exceed 1 cm. The relationship of this magnitude creates difficulties in the modeling of fields of stray currents in networks formed by the underground pipe installation, which means that we would have to determine a more complex model that includes parameters of the field that is being investigated.

In the rest of the ground field is three-dimensional land, and an expression for the potential corresponds with the Laplace equation. The simulation model created with the aid of computer should include two main parts: 1. a simple serial connection of partial elements of tube and 2. Parameters of the remaining part, interconnected so as to preserve the value of the potential on the merging surface of the models. The potential $\varphi_1(x, y)$ on that surface changes only along the axis of the tube and in order to preserve its value the surface of metal must be simulated. To ensure the potential change along the axis of the tube, the metal surface should be divided into smaller elements of the same length, and to each element of length Δl the longitudinal resistance and conductivity should be added. Equipotential lines, which are distant from the axis of the tube more than $20R$, are similar to semicircles with centre at the normal point on the surface of the ground that also passes through the center of the tube at axis AA level at the ground level. In this way we can graphically observe the impact of parameters on equipotential forces. By adjusting the corresponding parameters their operation in the space can be significantly reduced.

4. CONCLUSION

The spreading of stray currents is unavoidable throughout the grounding system. Sources of interferences in electrical networks, including the stray currents, cannot be completely removed. Currents that flow through the grounding system in the underground electrical conductive installations create differences of potentials. These currents are very dangerous for all electrical circuits, so the definition of grounding is the only way to reduce the influence of stray currents and enhance the system of potential equalization. The values of voltages and currents along the radial groundings of different lengths can be determined according to the method proposed in this paper. The paper developed an analytical procedure and obtained the formulas that could be used to assess the influence of parameters of resistance, inductance, capacitance and output to the groundings, at the moment of occurrence of stray currents of higher harmonics. From the obtained results it can be concluded that the conductive grounding structure is not equipotential when Type A radial grounding electrodes have been installed.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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