



# Collisional Growth within the Solar System’s Primordial Planetesimal Disk and the Timing of the Giant Planet Instability

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## Abstract

The large-scale structure of the solar system has been shaped by a transient dynamical instability that may have been triggered by the interaction of the giant planets with a massive primordial disk of icy debris. In this work, we investigate the conditions under which this primordial disk could have coalesced into planets using analytic and numerical calculations. In particular, we perform numerical simulations of the solar system’s early dynamical evolution that account for the viscous stirring and collisional damping within the disk. We demonstrate that if collisional damping would have been sufficient to maintain a temperate velocity dispersion, Earth-mass trans-Neptunian planets could have emerged within a timescale of 10 Myr. Therefore, our results favor a scenario wherein the dynamical instability of the outer solar system began immediately upon the dissipation of the gaseous nebula to avoid the overproduction of Earth-mass planets in the outer solar system.

*Unified Astronomy Thesaurus concepts:* [Solar system formation \(1530\)](#); [Astrodynamics \(76\)](#); [Trans-Neptunian objects \(1705\)](#)

## 1. Introduction

The emergent picture of the evolution of the early solar system is an instability-driven scenario. Tsiganis et al. (2005) proposed what is commonly referred to as the “Nice model,” in which the giant planets formed on circular, co-planar orbits with an accompanying planetesimal disk located between  $\sim 15$  and 35 au. Subsequent interactions with this primordial disk triggered a dynamical instability.

The Nice model reproduces a variety of characteristics of the present-day solar system, including the current orbits of the giant planets, the inclination distribution of the co-orbital Jupiter Trojans (Morbidelli et al. 2005; Nesvorný et al. 2013) and the existence and structure of the Kuiper Belt (Levison et al. 2008; Nesvorný & Morbidelli 2012; Nesvorný 2015; Gomes et al. 2018). For a recent review of the early evolution of the solar system, we refer the reader to Nesvorný (2018).

Although the Nice model successfully accounts for the large-scale structure of the solar system, numerous details remain elusive. Particularly, the precise timing of the instability is somewhat unconstrained. A promising avenue to constrain the timing of the instability was to relate it to the Late Heavy Bombardment (LHB) of the moon. The petrological record of Lunar craters implies that the Moon experienced a bombardment flux of planetesimals, roughly  $\sim 700$  Myr after the planets formed (Hartmann et al. 2000). It is uncertain whether this bombardment occurred during a cataclysmic spike or one that slowly decayed over time. This increase in cratering events could be caused by an increase of the bombardment rate (Tera et al. 1974; Ryder 1990, 2002). Gomes et al. (2005) demonstrated that the migration of the giant planets would naturally produce a sudden flux of planetesimals into the inner solar system, which would explain the LHB. Levison et al. (2011), however, demonstrated that the exchange of energy between the giant planets and the planetesimal disk would only explain the timing of the LHB if the disk’s inner edge was sufficiently removed from the orbit of Neptune. Alternatively, the LHB could also be explained by impacts from

leftover planetesimals at the tail-end of terrestrial planet accretion (Hartmann 1975; Neukum et al. 2001; Hartmann 2003).

In a set of recent studies, numerous authors have proposed that the Nice model instability and the LHB are unrelated and have argued for an early instability that started  $< 100$  Myr after the formation of the Sun (de Sousa et al. 2020; Nesvorný et al. 2021). Nesvorný et al. (2018) argued that the existence of the binary Jupiter Trojan (617) Patroclus-Menoetius (Grav et al. 2011; Buie et al. 2015) implied that the primordial disk dissipated via migrating planets within  $\sim 100$  Myr of the formation of the Sun. Beyond these concerns, aspects of the terrestrial planets are more readily reproduced with the early instability scenario. In particular, the survival of the terrestrial planets is more likely in the early instability model (Clement et al. 2018, 2019).

In this work, we study a related but distinct aspect of the instability-driven scenario: the evolution of the planetesimal disk. For computational purposes, most studies ignore self-gravity and thus the possibility of growth, generally in the planetesimal swarm. Accordingly, the possibility that the planetesimal disk could coalesce into planets has not been explored in detail. In this work, we ask a simple question: if the Sun was encircled by  $\sim 20 M_{\oplus}$  of debris for millions of years, could these debris have formed planets? We answer this question through analytic estimates and direct  $N$ -body simulations. The remainder of this Letter is organized as follows. In Section 2, we consider estimates of planetesimal growth rates and the excitation of their velocity dispersion from analytic grounds. In Section 3, we report the results of our numerical experiments, and discuss the implications of our results in Section 4.

## 2. Analytic Estimates

In this section, we consider accretion within the solar system’s planetesimal disk from analytic grounds. In particular, the goal of the following analysis is to obtain an estimate for

the timescale of mass growth that is expected to unfold in the primordial trans-Neptunian region of the solar system, and highlight its connection to the velocity dispersion.

The characteristics of the debris belt that drives planetary migration in the Nice model are readily summarized. The disk is typically assumed to be comprised of solids with a mass,  $M_{\text{disk}} \sim 20 M_{\oplus}$ , and to extend from  $r_0 \sim 15$  au to  $r_{\text{out}} \sim 35$  au. Moreover, the disk is envisioned to emerge from the Solar nebula in a dynamically cold state, meaning that the planetesimals that comprise this belt originate on roughly circular and co-planar orbits. For simplicity, here we adopt the surface density profile proposed in Mestel (1963), with  $\Sigma = \Sigma_0(r_0/r)$ , where  $\Sigma_0$  is the planetesimal surface density at the disk's inner edge and  $r$  is the radial distance. For simplicity, we assume that the velocity dispersion of the planetesimal belt is initially considerably smaller than the planetesimals' escape velocity, such that the Safronov number,  $\Theta = (v_{\text{esc}}^2/\langle v \rangle^2)$ , satisfies  $\Theta \gg 1$ . Here,  $v_{\text{esc}}$  and  $\langle v \rangle$  are the escape velocity and velocity dispersion of the planetesimals. The latter assumption is optimistic, and implies that the initial growth is predominantly facilitated by gravitational focusing (Safronov 1972).

Generically speaking, planetesimal growth can proceed via pebble accretion (Ormel et al. 2010; Lambrechts & Johansen 2012) or through pairwise collisions (Lissauer 1993). Although the former process is nominally faster, it is facilitated by the existence of the gaseous disk and does not operate in the absence of hydrodynamical drag forces. In this work, we are specifically concerned with post-nebular growth of planetesimals. Therefore, we ignore the effects of pebble accretion altogether and focus entirely on mutual impact driven assembly. The collisional mass-accretion rate of planetesimals,  $\dot{M}$ , can be estimated via an  $n - \sigma - \langle v \rangle$  relation.

The product of the typical planetesimal mass,  $m$ , and number density,  $n$  is given by  $mn \sim \Sigma/h$ , where  $h$  is the characteristic scale height of the planetesimal disk. The velocity dispersion can be written as  $\langle v \rangle \sim v_{\text{kep}}(r)(h/r)$ , where  $v_{\text{kep}}(r)$  is the Keplerian velocity at a given radial distance,  $r$ . The characteristic accretion rate,  $\dot{M}$ , of a planetesimal is given by

$$\dot{M} = 4 \pi \rho R^2 \frac{dR}{dt} \sim \Sigma \pi R^2 (1 + \Theta) \left( \frac{v_{\text{kep}}(r)}{r} \right), \quad (1)$$

where  $\rho$  is the density of the planetesimal. The gravitational focusing is accounted for by the  $(1 + \Theta)$  enhancement of the collisional cross section. The derivation of Equation (1) is analogous to the formalism developed in Armitage (2010). This expression yields a constant  $dR/dt$ , because  $\dot{M} \propto r^3$ . This can be recast as the characteristic timescale,  $\tau_{\text{accr}}$ , that is necessary for the radius to increase by some value  $\Delta R$ ,

$$\tau_{\text{accr}} \sim \frac{4 \rho \Delta R}{\Sigma \Omega (1 + \Theta)}. \quad (2)$$

We can estimate the value of the surface density using  $M_{\text{disk}} \sim 2 \pi r_0 \Sigma_0 (r_{\text{out}} - r_0)$ , where  $r_0$  and  $r_{\text{out}}$  are the inner and outer disk radii, respectively. We obtain  $\tau_{\text{accr}} \sim 10^{10}/\Theta$  yr for  $\Delta R \sim 10^3$  km,  $\rho \sim 2$  g cc<sup>-1</sup> and  $\Theta \gg 1$ .

Equation (2) highlights the fact that significant growth within the planetesimal disk can take place on a  $\sim 100$  Myr timescale *only* if the disk remains dynamically cold throughout this time. The ratio of the escape velocity to orbital velocity is on the order of  $v_{\text{esc}}/v_{\text{kep}} \sim 0.01$  for an  $R \sim 100$  km object orbiting at  $r_0$ . Therefore, the estimate above implies that sustained growth

requires eccentricities and inclinations smaller than  $\sim 1\%$ . In reality, the velocity dispersion of the planetesimal disk is controlled by an assortment of factors, including self-gravitational (viscous) stirring, collisional damping, and the system's approach toward equipartition.

To estimate the degree of orbital crossing that is expected to develop within the disk, we consider the competing effects of collisional damping and viscous stirring. For this order of magnitude calculation, we neglect planet-driven evolution and accretion, such that the disk is comprised of equal-mass bodies. In this regime, equipartition (which drives dynamical cooling of massive objects at the expense of dynamical heating of low-mass planetesimals) is inconsequential and all objects are characterized by a common velocity dispersion.

The evolution of the velocity dispersion is given by Armitage (2010), and can be estimated as

$$\frac{d\langle v \rangle}{dt} = \frac{2 \pi G^2 m \Sigma \Omega \ln(\Lambda)}{\langle v \rangle^3} - \frac{\langle v \rangle}{t_{\text{col}}}, \quad (3)$$

where  $m$  is the planetesimal mass, and  $\ln(\Lambda) \sim 10$  is the Coulomb logarithm. As above, the collisional timescale is given by the  $n - \sigma - \langle v \rangle$  relation, and takes the form

$$t_{\text{col}} = \frac{1}{n \pi R^2 \langle v \rangle} = \frac{2 m}{\pi R^2 \Sigma \Omega}. \quad (4)$$

In the absence of collisional damping, Equation (3) dictates that the velocity dispersion grows as  $\langle v \rangle \propto t^{1/4}$ . For finite  $t_{\text{col}}$ , however, there exists an equilibrium solution. Noting the approximate relationship,  $\langle v \rangle \sim e v_{\text{kep}}$ , this equilibrium corresponds to a characteristic eccentricity given by

$$\langle e \rangle \sim \left( \frac{2 \pi \ln(\Lambda) G^2 m \Sigma \Omega t_{\text{col}}}{v_{\text{kep}}^4} \right)^{1/4} \lesssim 0.02, \quad (5)$$

for an  $R \sim 100$  km body, which is the typical planetesimal size produced by the streaming instability (Youdin & Goodman 2005; Johansen et al. 2007). This expression provides a useful gauge for the effective dynamical temperature of the disk. It is important to note that this estimate does not account for planetary perturbations (which will stir the planetesimal swarm further), or dynamical friction (which will facilitate accretion of larger embryos, as is typical for simulations of oligarchic growth). Generally, these processes cannot be quantified analytically. Because our analytical estimates are conservative and do not account for gravitational focusing, in the next section we use numerical simulations to model trans-Neptunian accretion in the primordial solar system.

### 3. Numerical Experiments

To simulate accretion within the planetesimal disk, we used the `mercury6` code (Chambers 1999). Our numerical experiments follow the standard version of the Nice model. Planets were initialized in a compact multi-resonant configuration (Batygin & Brown 2010), encircled by a  $20M_{\oplus}$  disk of planetesimals. The disk's inner and outer edges were set to 15 au and 35 au, respectively. The calculations were performed using the hybrid Wisdom–Holman/Bulirsch–Stoer algorithm (Wisdom & Holman 1991). The timestep,  $\Delta t$  and accuracy parameter,  $\epsilon$ , were set to  $\Delta t = 300$  days and  $\epsilon = 10^{-12}$ , respectively.

We modeled the planetesimal disk itself with  $N_s = 1000$  super-planetesimals (to represent the larger population of small planetesimals within the disk), and seeded it with  $N_b = 100$  protoplanetary embryos (assumed to be fully formed at the start of the simulation and can accrete mass). All planetesimal and embryo masses were set to a common value at the beginning of the simulation,  $10^{-8}$  and  $10^{-7} M_\odot$  with radii  $\sim 2000$  km and  $\sim 5200$  km for fiducial small and large seeds, respectively. All particles were initialized with negligible eccentricities and inclinations. Nesvorný et al. (2020) presented a methodology to assign radii to superparticles such that numerical simulations have the correct collisional timescale. Here, we assume that each simulated protoplanetary embryo represents a fully formed, spherical particle with a bulk density of  $1 \text{ g cm}^{-3}$ . While this is an appropriate approximation for the accreting protoplanetary embryos, it is not necessarily appropriate for the superparticles, which represent a larger number of planetesimals. We performed convergence tests with initial masses of superparticles that were  $\sim 1/2$  and  $\sim 2\times$  the initial mass, and verified that the results of the simulations did not sensitively depend on the initial radii of the particles. While the interactions among the embryos and planets were self-consistently modeled in a conventional  $N$ -body fashion, self-gravitational coupling among the planetesimals was neglected to conserve computational costs. All collisions were treated as perfect mergers.

Despite being a necessary approximation for computational efficiency, the super-particle modeling scheme leads to a strongly over-excited velocity dispersion. This is because a vast population of undamped massive planetesimals generates much stronger gravitational scattering events than those that occurred in the real disk. This leads to an unphysical excitation of the distribution of the planetesimals' eccentricities and inclinations. To counteract this effect in our calculation, we mimicked the effects of collisional damping and dynamical friction by introducing a fictitious acceleration,  $\vec{a}_{\text{damp}}$ , into the equations of motion at each timestep (Papaloizou & Larwood 2000) of the form

$$\vec{a}_{\text{damp}} = -2 \left( \frac{\vec{v} \cdot \vec{r}}{r^2 \tau} \right) \hat{r} - 2 \left( \frac{\vec{v} \cdot \vec{k}}{\tau} \right) \hat{k}, \quad (6)$$

where  $\vec{k} = (0, 0, z)$  is the vector corresponding to the  $z$ -component of the planetesimals' position.

Simply put, this acceleration does not affect objects that occupy circular and planar orbits, but damps the eccentricities and inclinations on a characteristic timescale,  $\tau$ , if they develop. To calibrate the damping time in our numerical experiments, we tuned the value of  $\tau$  until our simulation reproduced the viscous stirring—collisional damping equilibrium discussed in the previous section. We did this by removing the giant planets from the simulation and suppressing physical collisions within the disk such that the planetesimals and embryos retain equal masses in perpetuity.

Varying  $\tau$ , we carried out each test integration for  $\sim 10$  damping times, and recovered the  $\langle e \rangle \propto \tau^{1/4}$  relationship predicted by Equation (3). Accordingly, we found that a damping time of  $\tau = 50,000 \text{ yr}$  yields  $\langle e \rangle = 0.014$ —a value that is close to our analytic estimate (see Table 1 and the discussion in the final paragraph of this section). This result is consistent with the numerical results presented by Levison et al. (2011), who carried out detailed simulations of the early dynamical evolution of the solar system, and also found that

**Table 1**  
Median Eccentricity and Inclination in the Numerical Simulations as a Function of Damping Time,  $\tau$

Damping Time	Median Eccentricity	Median Inclination
$2 \times 10^4 \text{ yr}$	0.011	$0.20^\circ$
$3 \times 10^4 \text{ yr}$	0.013	$0.31^\circ$
$5 \times 10^4 \text{ yr}$	0.014	$0.31^\circ$
$1 \times 10^5 \text{ yr}$	0.018	$0.46^\circ$

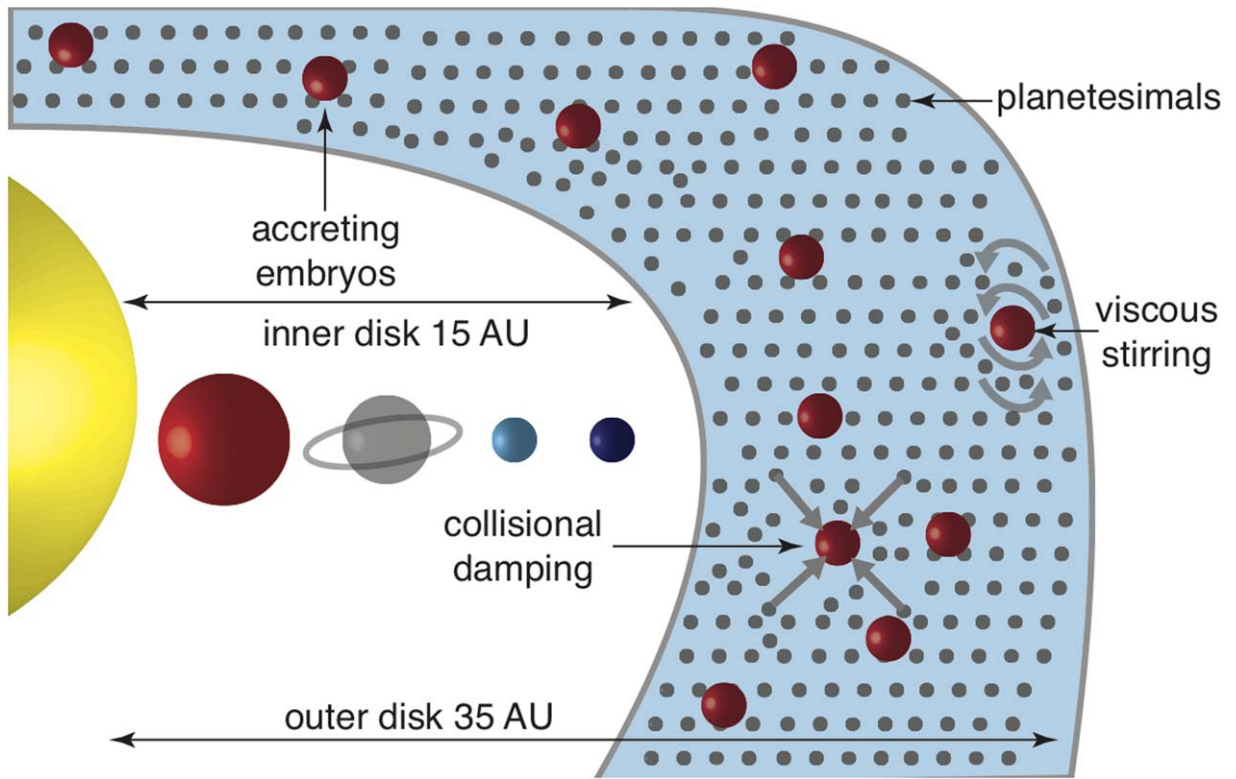
viscous stirring yields rms disk eccentricity of approximately 2%. Adjusting the gravitational potential length scale for each particle could provide an alternative solution to damping the over-excitation of particles.

With our numerical setup calibrated, we restored the giant planets into the simulation and evolved the solar system, fully accounting for growth of protoplanetary embryos through pairwise impacts. In Figure 1, we show a schematic of the initial conditions of and relevant processes included in the simulation. Intriguingly, our simulations indicate that if the planetary instability or rapid outward migration of Neptune does not ensue shortly after the dissipation of the protosolar nebula, then significant collisional growth of embryos unfolds on a  $\sim 10$  Myr timescale.

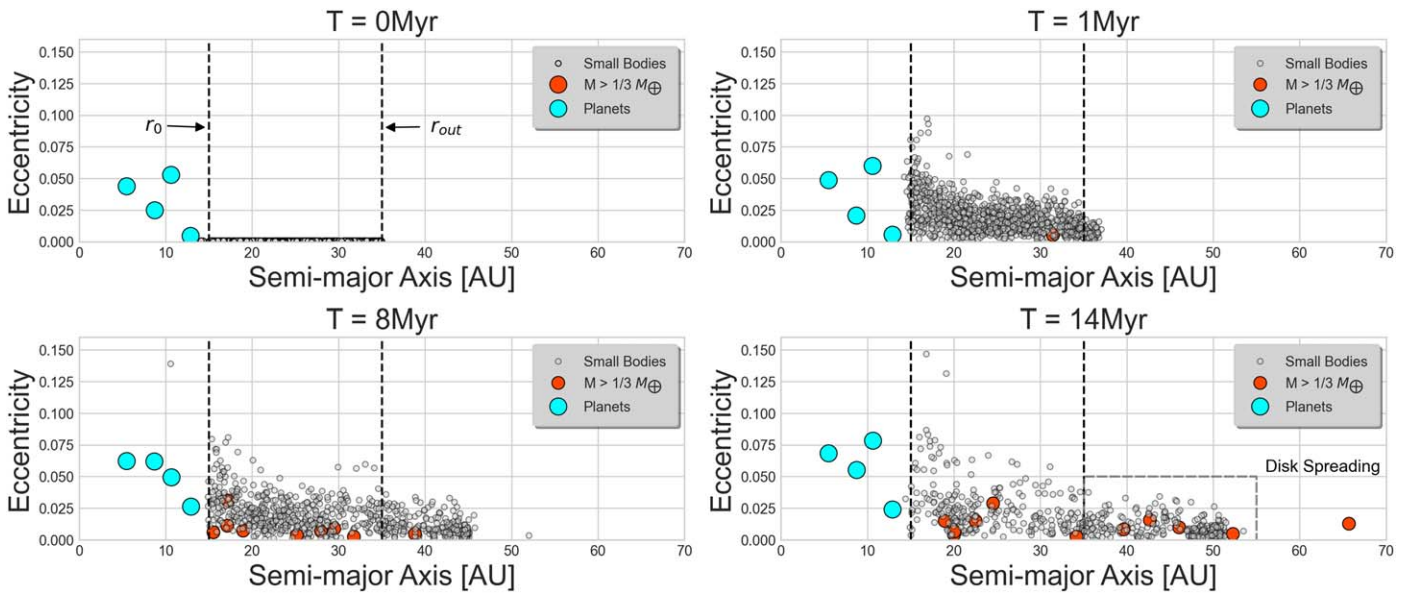
A series of snapshots of the dynamical state of the outer solar system attained within our fiducial simulation are shown in Figures 2 and 3. In some similarity with standard models of terrestrial planet accretion (Chambers 2001; Raymond et al. 2011), our numerical experiment shows how the initially circular and planar disk of planetesimals begins to coalesce, and because of dynamical friction, growing embryos settle to the midplane where their continued accretion is further aided by a diminished velocity dispersion. Within  $\sim 10$  Myr, numerous  $\sim M_\oplus$  planets emerge within the disk, and the system eventually enters a slower mode of accretion. The mass evolution of the accreting embryos within our model disk is shown in Figure 4. Of the 100 initial embryos in the simulation, two grow to larger than  $\sim 1 M_\oplus$  and  $\sim 10\%$  of the objects grew to masses of  $m \in (0.25, 1) M_\oplus$ .

As seen in Figures 2 and 3, the planetesimal disk spreads beyond the initial edge limit of 35 au. Under the assumption that the disk spread beyond 35 au, Neptune would not stop migrating at 30 au, as it would still have the ability to undergo planetesimal driven migration. However, the outer regions of the disk that spread viscously is not dynamically cold, and could potentially violate the conditions necessary for planetesimal driven migration. Moreover, as depicted in Figure 2 and Figure 3, large planetary embryos formed within the disk by  $T = 8$  Myr. Clearly, we do not see either of these features represented in the structure of the Kuiper Belt or Neptune's current orbit. In order to alleviate this discrepancy, Neptune would have had to have undergone rapid planetesimal driven migration. This would eliminate the opportunity for large planetary embryos to form within the disk. Therefore, we conclude that the instability must have occurred very early in the formation of the solar system.

It is crucial to keep in mind that our results are contingent upon the assertion that the competition between viscous stirring and collisional damping prevents the disk's eccentricity and inclination distributions from widening too much. We verified that in the absence of damping, no collisional growth ensues in our calculation. Because the effects of the disk's dynamical friction are implemented within our model through a fictitious damping acceleration, this effect is governed by our choice of a



**Figure 1.** Schematic diagram depicting the initial conditions and components of the numerical simulations in Section 3. The integrations have the four giant planets at their primordial locations. The primordial disk is composed of passive planetesimals and accreting embryos, and is located between 15 and 35 au. The effects of viscous stirring and collisional damping, discussed in Section 2, are implemented throughout the simulation.



**Figure 2.** Evolution of eccentricities and semimajor axes for all of the objects in the numerical simulation. Small bodies, roughly  $\sim 1M_{\oplus}$  bodies and giant planets are indicated by gray, red, and blue points, respectively. The four panels show the orbital elements at times corresponding to 0, 1, 8, and 14 Myr in the simulation.

finite value for  $\tau$ . As already noted above, however, the  $\tau \rightarrow \infty$  assumption leads to an unphysically over-excited disk, and therefore significantly underestimates the potential for collisional growth within the system. We further note that the scaling between  $\langle e \rangle$  and  $\tau$  is rather forgiving, such that choosing a somewhat higher value for the damping timescale in our model should yield similar results.

To verify this, we ran analogous simulations with four different damping timescales,  $\tau$ , and calculated the median eccentricity and inclination for all of the planetesimals after the disk reached an equilibrium state,  $\sim 1$  Myr for all cases. The results of these experiments are shown in Table 1. Evidently, the median eccentricity and inclination do not sensitively depend on the magnitude of  $\tau$ .

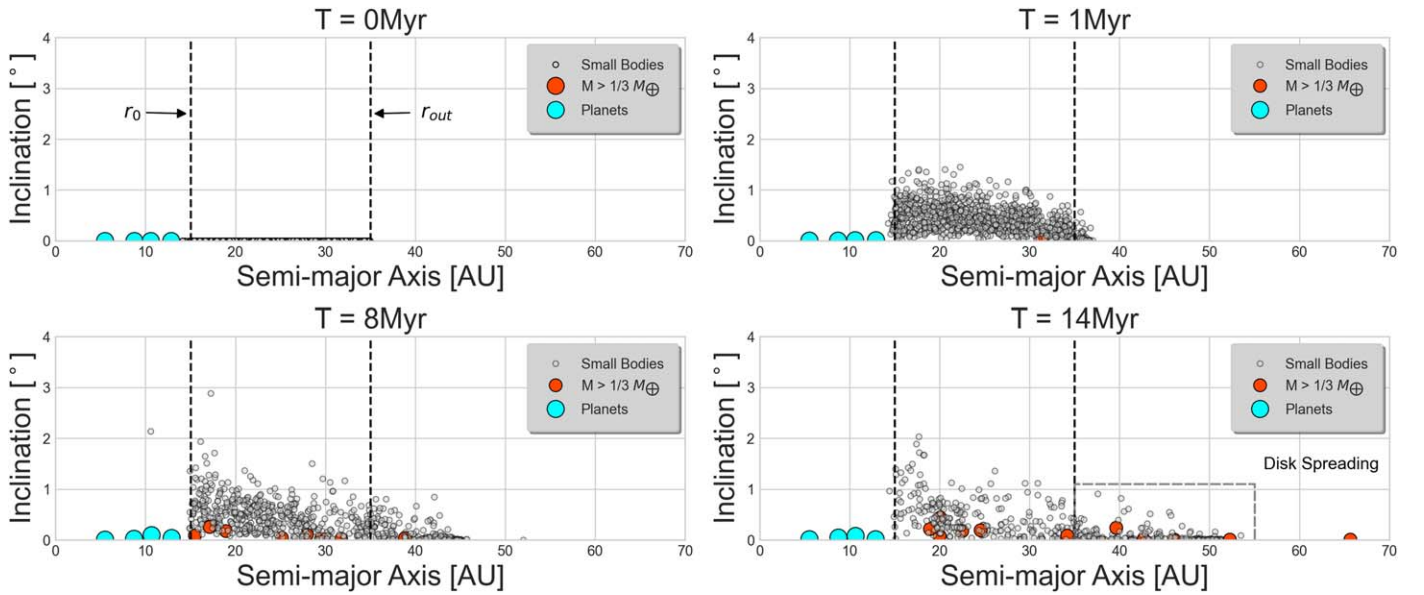


Figure 3. Same as in Figure 2, for inclinations and semimajor axes.

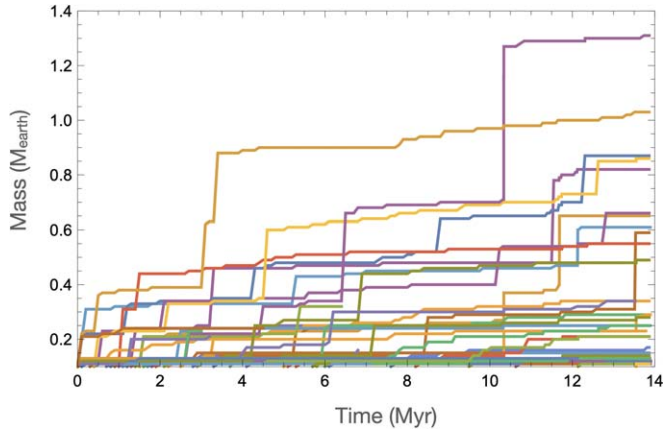


Figure 4. Mass evolution of the 100 accreting embryos in the simulation. The oligarchic growth within the disk occurs on timescales of order  $\tau \sim 10$  Myr.

#### 4. Discussion

In this work, we explored the possibility of post-nebular oligarchic growth of protoplanetary embryos in the solar system’s primordial disk of debris. Overall, our numerical experiments indicate that it is plausible for  $\sim 1M_{\oplus}$  objects to emerge within the ancient trans-Neptunian region (i.e.,  $\sim 15$ – $35$  au) on a  $\sim 10$  Myr timescale, provided that the disk’s velocity dispersion remains temperate. Accordingly, our numerical experiments indicate that if the planetesimal disk were to remain unperturbed for  $\sim 10$ – $100$  Myr, the present-day architecture of the solar system likely would have been very different.

In virtually all respects of the physical setup, our simulations are similar to standard realizations of the Nice model (Levison et al. 2008, 2011; Batygin et al. 2011; Nesvorný & Morbidelli 2012). Previously published self-gravitating  $N$ -body simulations of early dynamical instabilities in the solar system had interactions among superparticles that led to an unphysical degree of self-excitation within the planetesimal disk (Reyes-Ruiz et al. 2015; Fan & Batygin 2017). In our simulations, we introduced dissipation within the disk to approximately capture the effects of collisional damping. This effect maintains the planetesimals’

eccentricities and inclinations on the order of a few percent and few degrees, respectively, and facilitates accretion of protoplanetary embryos. Moreover, in this scenario, gravitational scattering forces a substantial amount of icy debris to spread beyond the present-day orbit of Neptune.

Cumulatively, our results translate to an intriguing constraint on the instability timescale. That is, if the solar system’s transient phase of planet–planet scattering were to be delayed by hundreds of millions of years as envisioned in the original version of the Nice model (Gomes et al. 2005), the disk would have coalesced into planets, compromising its capacity to serve as the trigger mechanism for the instability. Consequently, our results suggest that the outward migration of Neptune would have started shortly after the dissipation of the protoplanetary nebula. This result is consistent with the conclusions of recent work that favored the early instability scenario (Clement et al. 2018; Nesvorný et al. 2018; Clement et al. 2019; de Sousa et al. 2020; Nesvorný et al. 2021).

There are numerous ways to expand upon our study. A limitation of our calculation is the relatively small number of particles in the simulation. Numerical methods such as those implemented in the GENGA code (Grimm & Stadel 2014), which uses GPUs to calculate gravitational interactions between bodies and produces results consistent with `mercury6` (Chambers 1999), could be used to expand these simulations, such as in Quarles & Kaib (2019).




We have also ignored the initial size distribution of particles, although Kaib et al. (2021) found that in order to explain the existence of Pluto and Eris, the number of  $\sim$ Pluto-mass bodies in the primordial Kuiper Belt would have been as few as  $\sim 200$  and no greater than  $\sim 1000$ . Shannon & Dawson (2018) provided similar limits on the number of primordial  $\sim$ Pluto-mass and  $\sim$ Earth-mass protoplanets by considering the survival of ultra-wide binary trans-Neptunian objects, the Cold Classical Kuiper Belt, and the resonant population. If these large planetesimals formed during the gaseous disk stage and were present in the primordial disk, this would likely speed up accretion such that Earth-mass objects would emerge on even shorter timescales via the same physical processes explored in our simulations. If instead these objects formed after the disk

dispersal, then the simulations described in the previous paragraph could test this hypothesis. For computational efficiency, our simulations used accreting embryos that are slightly larger than Pluto initially. We expect that Pluto-mass objects could have formed at some  $t < 10$  Myr based on our collisional growth calculations.

In any case, considerations of growth in the disk provide an intriguing possibility to further constrain the instability-driven scenario of the early evolution of the solar system.

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