



Modulated Feedback and Coupling Time Delays, and All-to-all Chaos Synchronisation in a Network of Networks: One of the Simplest Cases

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Authors' contributions

This work was carried out in collaboration between all authors. Author EMS designed the study, wrote the algorithm, methodology of programming, interpreted the results and wrote the first draft of the manuscript. Authors PAB, RAN and LHH managed the analyses of the study. Author MVQ managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

The study reports on all- to- all chaos synchronisation in a network of networks based on the Ikeda model. The study considered one of the simplest cases. It found the existence and stability conditions for such a synchronisation regime. Numerical simulations validated the analytical findings. The results can be of certain importance in achieving high- level output for the coupled systems and information processing.

Keywords: *Network of networks; Ikeda model; time-delay systems; modulated feedback and coupling delay times; all-to-all chaos synchronisation; existence and stability conditions.*

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1. INTRODUCTION

Networks or a network of networks is a widespread concept in a world-wide-web, population dynamics, neuroscience, power grids, communication, social and computer systems, etc. Research of such interacting systems is a very hot topic in nonlinear dynamics, see, e.g. [1-6] and references therein.

Chaos synchronisation [1] as a control method is of fundamental importance in a variety of complex physical, chemical and biological systems [7]. Synchronisation of chaos refers to a process wherein two (or many) chaotic systems (either equivalent or nonequivalent) adjust a given property of their motion to a common behaviour due to a coupling or to a forcing (periodical or noisy) [7]. In the context of coupled chaotic elements, many different synchronisation states have been studied, namely complete or identical synchronisation, phase synchronisation, lag synchronisation, generalised synchronisation, anticipating synchronisation, etc. [7-9]. Complete synchronisation [10] was the first to be discovered and is the simplest form of synchronisation in chaotic systems. It consists in perfect hooking of the chaotic trajectories of two systems which is achieved using a coupling signal, in such a way that they remain in step with each other in the course of the time. Generalised synchronisation [11] goes further in using completely different systems and associating the output of one system to a given function of the output of the other system. Coupled nonidentical oscillatory or rotatory systems can reach an intermediate regime of phase synchronisation [12-14], wherein locking of phases occurs, while correlation in the amplitudes remains weak. Lag synchronisation [15] is a step between phase synchronisation and complete synchronisation. It implies the asymptotic boundedness of the difference between the output of one system at time t and the output of the other shifted in time (lag time). This implies that the two outputs lock their phases and amplitudes, but with the presence of a time lag. In anticipating synchronisation [16-17] the driven system state is synchronised to the future state of the driver system. For some other types of synchronisation see other references [18-21] also.

Synchronisation in complex systems is of a certain importance in governing and performance improving point of view, e.g. enhancing emission power from such systems [7]. Additionally, from

the fundamental point of view synchronisation of coupled (chaotic) systems eliminates some degrees of freedom of the coupled system and so produces a significant reduction of complexity, thus allowing for significant simplification of computational and theoretical analysis of the system.

As synchronisation in a wider sense is associated with communication, a study of existence and stability conditions for synchronisation is of paramount importance in networks. Synchronisation is important in chaos-based communication system to decode the transmitted message [7,17]: At the transmitter part of the communication system a message is masked with chaos, then chaos masked message is transmitted to the receiver system. At the receiver part of the communication system due to the chaos synchronisation between the transmitter and the receiver systems chaos is regenerated. Finally, deducting the receiver input and the receiver output one can decode the transmitted message, (as shown in Fig. 1).

This paper studies chaos synchronisation in one of the simplest cases of the network of networks based on the Ikeda system-paradigmatic model of chaotic dynamics in time- delay systems [22]. In case of constant time delays, analytically the existence and sufficient stability conditions for complete synchronisation between all the constituents of the network were derived. This supports the analytical findings with the numerical simulations. This paper also present example of chaos synchronisation between the constituent Ikeda models in case of variable time delay systems.

The organisation of the rest of this paper is as follows. In Sec. 2 introduction of the working model and the results of the analytical study have been presented. Section 3 is dedicated to the numerical simulations of all-to-all chaos synchronisation between the Ikeda models, including the case of modulated time delays. The results are summarised in Sec. 4.

2. SYSTEM MODEL

Consider all-to-all synchronisation between the chaotic Ikeda systems with the following coupling topology (see, Fig. 2): x -Ikeda system governs both networks (y, z) and (u, w) which consists of only two unidirectionally coupled Ikeda systems. For simplicity, consider the case when all the Ikeda systems are identical and time delays in the network are constant.

FIG.1

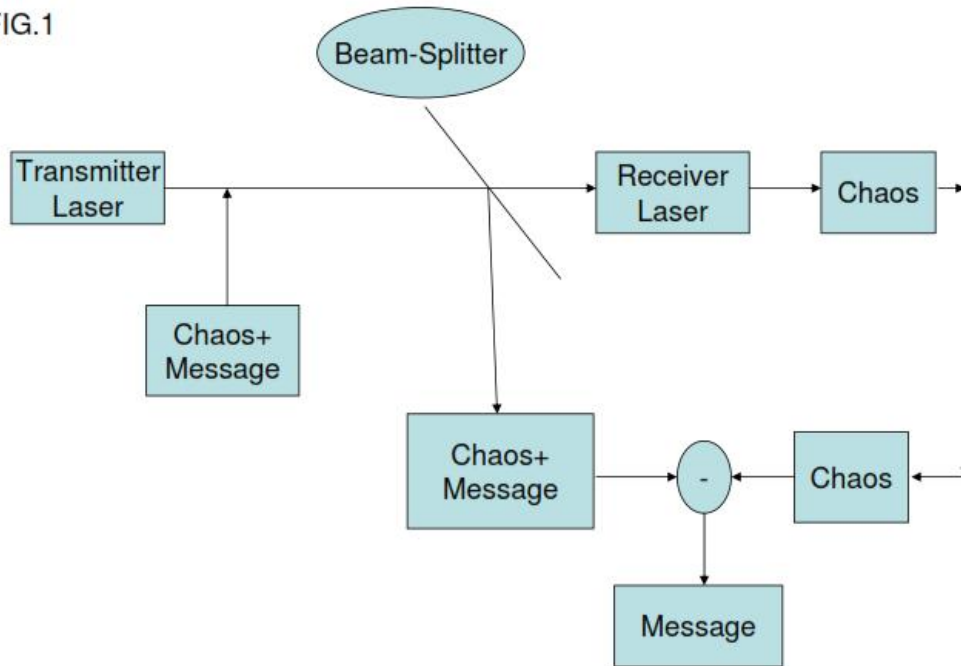


Fig. 1. Schematic view of chaos based communication system. For details, see, text

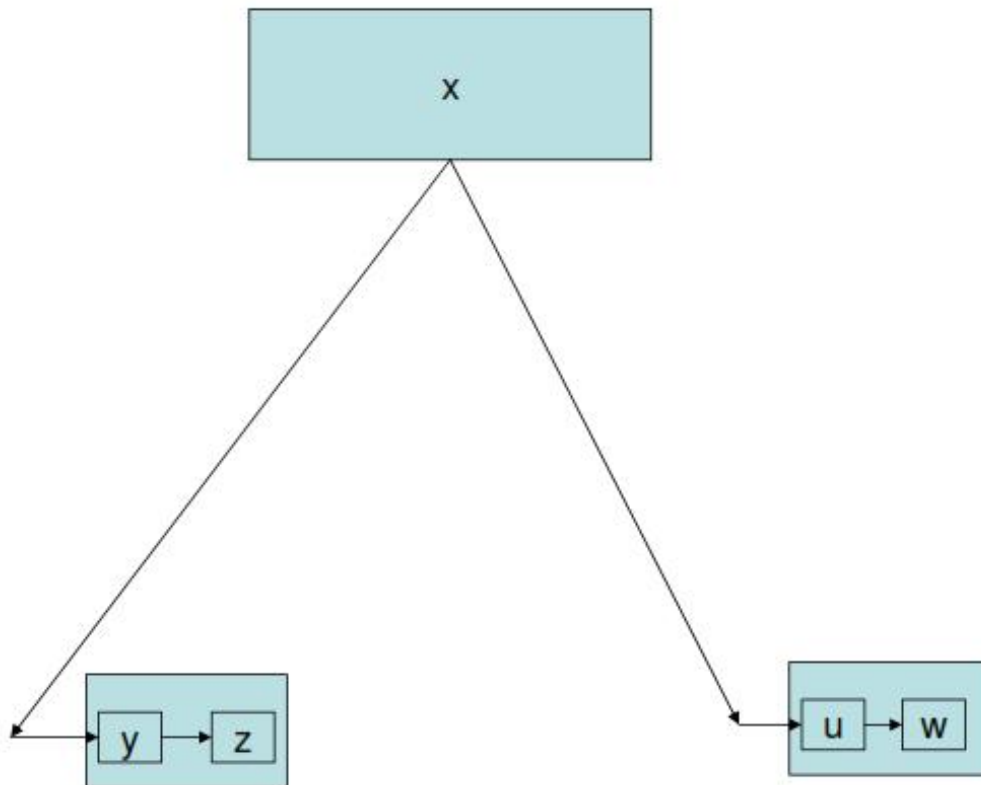


Fig. 2. Schematic view of the system under consideration, see text for details

$$\frac{dx}{dt} = -\alpha x + m_1 \sin x_{\tau} \quad (1)$$

$$\frac{dy}{dt} = -\alpha y + m_2 \sin y_{\tau} + m_6 \sin x_{\tau_1} \quad (2)$$

$$\frac{dz}{dt} = -\alpha z + m_3 \sin z_{\tau} + m_8 \sin y_{\tau_1} \quad (3)$$

$$\frac{du}{dt} = -\alpha u + m_4 \sin u_{\tau} + m_7 \sin x_{\tau_1} \quad (4)$$

$$\frac{dw}{dt} = -\alpha w + m_5 \sin w_{\tau} + m_9 \sin u_{\tau_1} \quad (5)$$

Here $x_{\tau} \equiv x(t - \tau)$. The same is valid for the other dynamical variables y, z, u, w .

Initially the Ikeda model was introduced to describe the dynamics of an optical bistable resonator, playing an important role in electronics and physiological studies and is well-known for delay-induced chaotic behaviour, see e.g. [22] and references therein. Later it was established that the Ikeda model or its modifications can be used to describe the dynamics of an opto-electronical, an acousto-optical systems and even the dynamics of the wavelength of the Distributed Bragg Reflector (DBR) Laser [22]. Furthermore, this investigation is of considerable practical importance, as the equations of the class B lasers with feedback (typical representatives of class B are solid-state, semiconductor, and low-pressure CO₂ lasers [23]) can be reduced to an equation of the Ikeda type [24].

Physically x is the phase lag of the electric field across the resonator (it should be noted that in the opto-electronical and acousto-optical systems x is proportional to the voltage fed to a modulator [12]); α is the relaxation coefficient for the driving x and driven y, z, u, w dynamical variables; τ is the feedback loop time delay;

τ_1 is the coupling time delay between x and y, y and z, x and u, u and w ; the case will be

$$m_1 = 2m_2; m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 \quad (11)$$

considered as $\tau = \tau_1$; m_1, m_2, m_3, m_4, m_5 are the feedback strengths for the Ikeda systems x, y, z, u, w respectively; m_6, m_8, m_7, m_9 are the coupling strengths between the systems x and y, y and z, x and u, u and w , respectively.

It is noted that system x is directly connected to the system y and connection to system z occurs via system y . Analogously, system x is directly connected to system u and connection to system w occurs via system u . It should also be emphasised that there is no direct connection between the networks (y, z) and (u, w) .

As mentioned above the all-to-all synchronisation was considered for the coupling topology presented in Fig. 2. Firstly, the complete synchronisation case between the variables x and y is considered. It is straightforward to establish that the synchronisation error $\Delta_{x,y} = x - y$ under the condition

$$m_2 = m_1 - m_6 \quad (6)$$

obeys the dynamics

$$\frac{d\Delta_{x,y}}{dt} = -\alpha\Delta_{x,y} + m_2\Delta_{x,y} \cos x_{\tau} \quad (7)$$

Obviously $\Delta_{x,y} = 0$ is a solution of system (7).

The sufficient stability condition of the synchronisation regime

$$x=y \quad (8)$$

can be found by applying the Lyapunov-Krasovskii functional approach [25-26]:

$$\alpha > |m_2| \quad (9)$$

By applying this procedure to synchronisation between the dynamical variables y and z, x and z, x and u, u and w, x and w, y and u, z and u, y and w, z and w the study establishes that for the configuration in Fig. 2 all-to-all complete synchronisation

$$x = y = z = u = w \quad (10)$$

occurs under the following conditions:

The formula (11) is the existence condition and formula (9) is the stability condition for all-to-all complete synchronisation (10).

In the next section the results of the numerical simulations of this synchronisation regime are presented.

3. NUMERICAL SIMULATIONS AND DISCUSSION

This section numerically demonstrates that how the analytical findings of the previous Section are validated. Synchronisation quality is characterised by the cross-correlation coefficient C [27] between the dynamical variables say x and y :

$$C(\Delta t) = \frac{\langle (x(t) - \langle x \rangle)(y(t + \Delta t) - \langle y \rangle) \rangle}{\sqrt{\langle (x(t) - \langle x \rangle)^2 \rangle \langle (y(t + \Delta t) - \langle y \rangle)^2 \rangle}} \quad (12)$$

$$C_{x,y} = C_{x,z} = C_{x,u} = C_{x,w} = C_{y,z} = C_{y,u} = C_{y,w} = C_{z,u} = C_{u,w} = 0.99.$$

where the brackets $\langle \cdot \rangle$ represent the time average; Δt is a time shift between the dynamical variables. In the present case $\Delta t = 0$. This coefficient indicates the quality of synchronisation: $C=1$ means perfect complete synchronisation.

Fig. 3 portrays time series of the system z for parameter values.

$\alpha = 8.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau = 5$.
 Fig. 4 presents synchronisation error dynamics $\Delta_{z,w} = z - w$ versus time for parameters as in Fig. 3. $C_{z,w} = 0.99$ is the cross-correlation coefficient between the systems z and w . For parameter values as in for Fig. 3 the other cross-correlation coefficients are

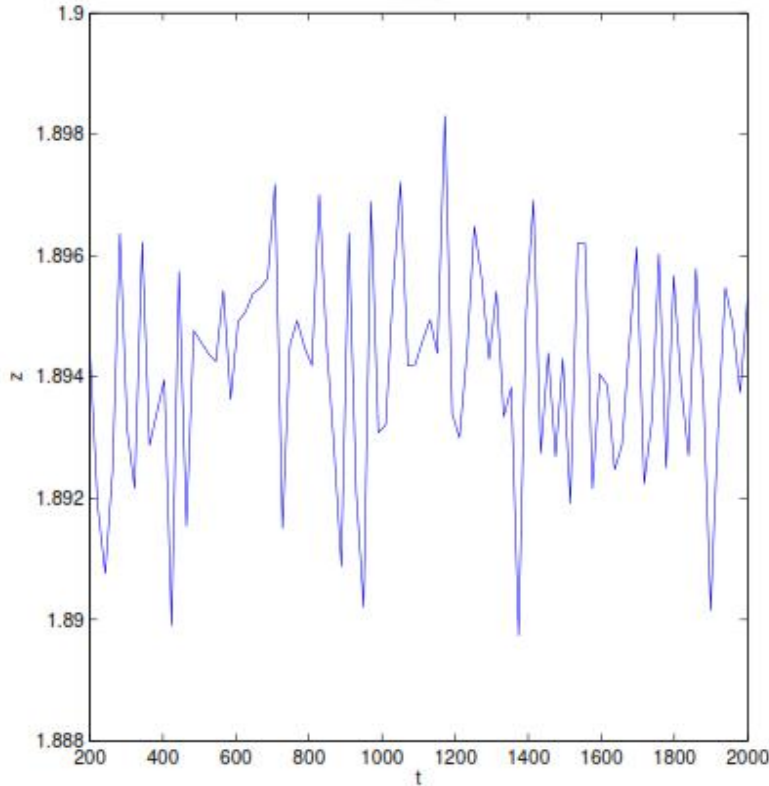


Fig. 3. Numerical simulation of all-to-all synchronization between Ikeda systems with the coupling scheme described in Fig.2, Eqs. (1-5) for

$$\alpha = 8.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau = 5.$$

Dynamics of the system z is shown. Dimensionless units

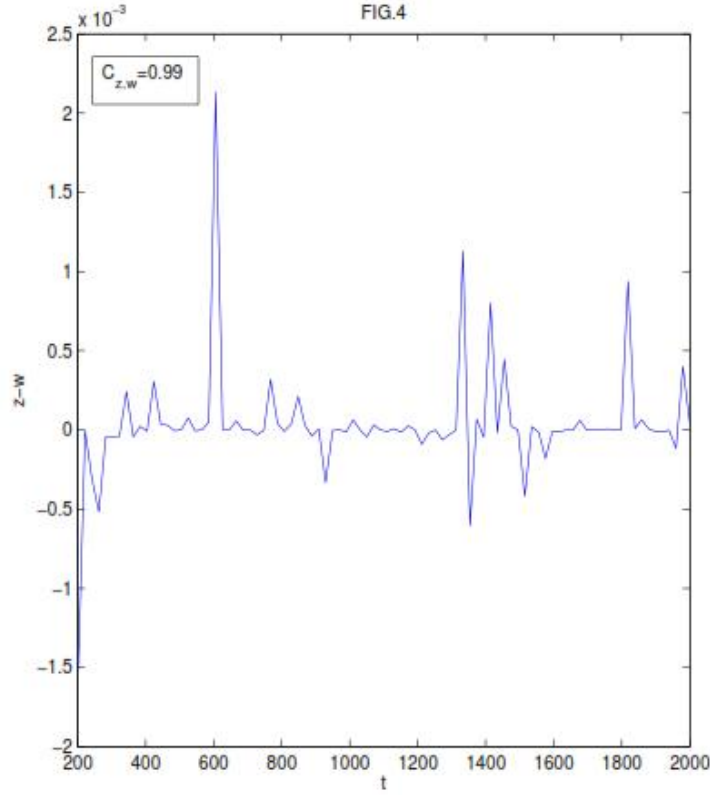


Fig. 4. Error dynamics $\Delta_{z,w} = z - w$ versus time t for parameters as in Fig. 3

$C_{z,w}$ is the cross-correlation coefficient between the systems z and w . Dimensionless units

The value of the cross-correlation coefficients testifies to the high-quality chaos synchronisation, which is vital for information processing in chaos-based communication systems and other possible applications. In numerical simulations, the synchronisation case between the outermost Ikeda models- z and w are mainly presented.

It should be noted that the approach based on the Lyapunov-Krasovskii method gives a sufficient stability condition for synchronisation, but does not forbid synchronisation [25] when the condition (9) is not met. In Fig. 5 and Fig. 6 the case of chaos synchronisation is presented when the stability condition for all-to-all synchronisation (9) is violated. Fig. 5 shows the dynamics of the system z for parameter values

$$\alpha = 3.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = \text{Er}$$

$$m_9 = 8, m_1 = 2m_2 = 16, \tau = 5.$$

ror $\Delta_{z,w} = z - w$ dynamics is presented in Fig. 6.

It is seen that despite the fact that condition (9) is violated, there is a high degree of synchronisation. $C_{z,w} = 1$ is the cross-correlation coefficient between the systems z and w . For this case the other cross-correlation coefficients are

$$C_{x,y} = C_{x,z} = C_{x,u} = C_{x,w} = C_{y,z} = C_{y,u} = C_{y,w} =$$

$$C_{z,u} = C_{u,w} = 1$$

It was noticed that larger values of the relaxation coefficient α decrease the amplitude of the chaotic vibrations. Comparing the dynamics of the variable z (Fig. 3 and Fig. 5) and the error $z-w$ dynamics (Fig. 4 and Fig. 6) one should pay attention to the scale on the ordinate axis.

Next, this study considers the case of variable time delays in the constituent Ikeda models, e.g. both the feedback and coupling time delays are variable. The role of modulated feedback and coupling time delays in controlling chaos in some laser systems was studied by Shahverdiev, 2016 [28].

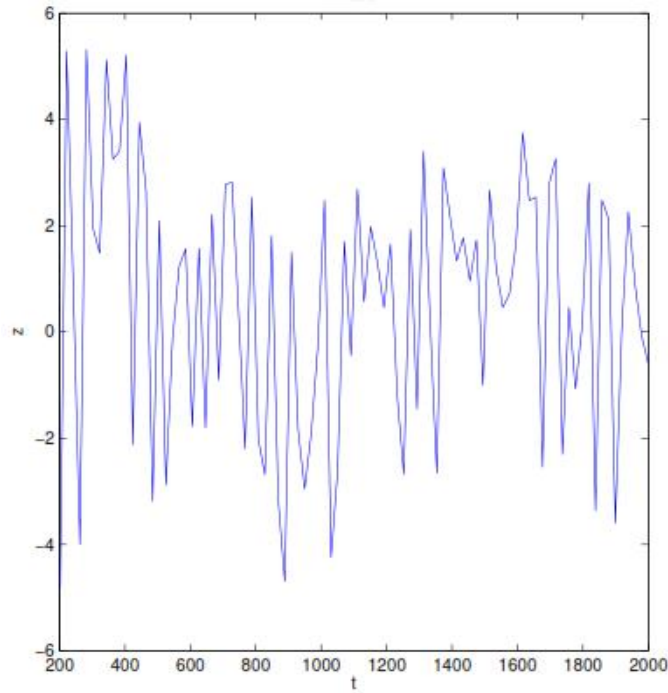


Fig. 5. Numerical simulation of all-to-all synchronization between Ikeda systems with the coupling scheme described in Fig. 2, Eqs. (1-5) for $\alpha = 3.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau = 5$. Note that stability condition (4) is not fulfilled. Time series of the system z is shown. Dimensionless units

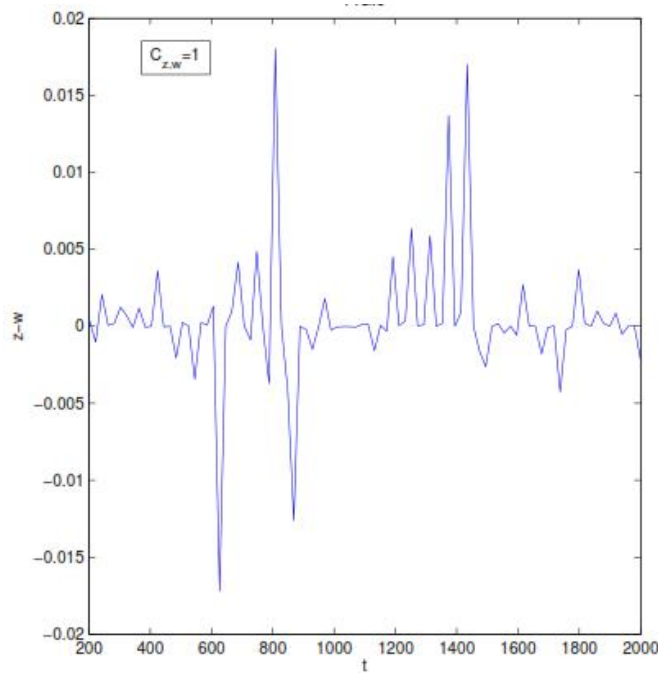


Fig. 6. Error dynamics $\Delta_{z,w} = z - w$ versus time t for parameters as in Fig. 5 $C_{z,w}$ is the cross-correlation coefficient between the systems z and w . Dimensionless units

This study considers three cases of time delay modulations: a) sinusoidal modulation of time delays; b) chaotic modulation of time delays; c) combined sinusoidal and chaotic modulation of time delays.

For sinusoidal modulations,

$$\tau(t) = \tau + \tau_a \sin(\omega_m t), \quad (13)$$

where τ is the zero-frequency component (constant time delay) τ_a is the amplitude, ω_m is the frequency of the modulation. For this case, following set of the new parameters is used: $\tau = 5$, $\tau_a = 1$, $\omega_m = 0.1$. Fig.7 shows the dynamics of the Ikeda model x . Numerical simulations show that for this case the correlation coefficients between the junctions are:

$$C_{x,y} = C_{x,z} = C_{x,u} = C_{x,w} = C_{y,z} = C_{y,u} = C_{y,w} = C_{z,u} = C_{u,w} = 1$$

Fig. 8 demonstrates highest quality synchronisation between Ikeda models z and w : Correlation coefficient $C_{z,w} = 1$. Fig. 8(a) pictures the dynamics of variables x (solid line) and y (dotted line) in one plot. It is clear that after some transient processes the dynamics of both variables coincide with each other. Correlation coefficient $C_{x,y} = 1$.

For the case of chaotic modulations of the coupling time delays the following form is chosen:

$$\tau(t) = 5 + 0.8x_1(t), \quad (14)$$

where $x_1(t)$ is the chaotic solution of the Ikeda model:

$$\frac{dx_1}{dt} = -2x_1(t) + 10 \sin x_1(t - 5). \quad (15)$$

Chaotic dynamics of x for parameters as in Eq.(15) and

$$\alpha = 3.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau = 5$$

is shown in Fig. 9. According to the numerical

simulations, for the case of chaotically modulated feedback and coupling time delays the correlation coefficients between the Ikeda models are:

$$C_{x,y} = C_{x,z} = C_{x,u} = C_{x,w} = C_{y,z} = C_{y,u} = C_{y,w} = C_{z,u} = C_{u,w} = C_{z,w} = 1$$

Finally, the case of the combined sinusoidal and chaotic modulations of the coupling time delays is considered as:

$$\tau(t) = 5 + 0.5x_1(t) \sin(0.1t). \quad (16)$$

The results of the numerical modelling for this case are:

$$C_{x,y} = C_{x,z} = C_{x,u} = C_{x,w} = C_{y,z} = C_{y,u} = C_{y,w} = C_{z,u} = C_{u,w} = C_{z,w} = 1$$

In support of high- quality synchronisation between the driven Ikeda models, in Fig. 10 dependence of z on w is demonstrated.

This study has also numerically experimented with different amplitudes and frequencies of the modulation and obtained that the synchronisation quality is quite robust to such modulations. As shown by the numerical simulations the effect of dithering coupling and feedback time delays on the synchronisation quality between the Ikeda models is not pronounced. In other words, the studied configuration of Ikeda models is quite robust to the modulation of the coupling and feedback delays.

Thus, these results testify that driven Ikeda models, although are not coupled directly between themselves, can be synchronised quite robustly by a single driver model even under the conditions of the dithered feedback and coupling time delays.

Complete synchronisation between two Ikeda models was investigated in previous work [29] where the authors considered the case of sinusoidal modulation of the feedback time delays. In this paper, complete synchronisation is considered under the modulation of both feedback and coupling time delays (including the case of chaotic modulation) in a network (however simple) Ikeda system.

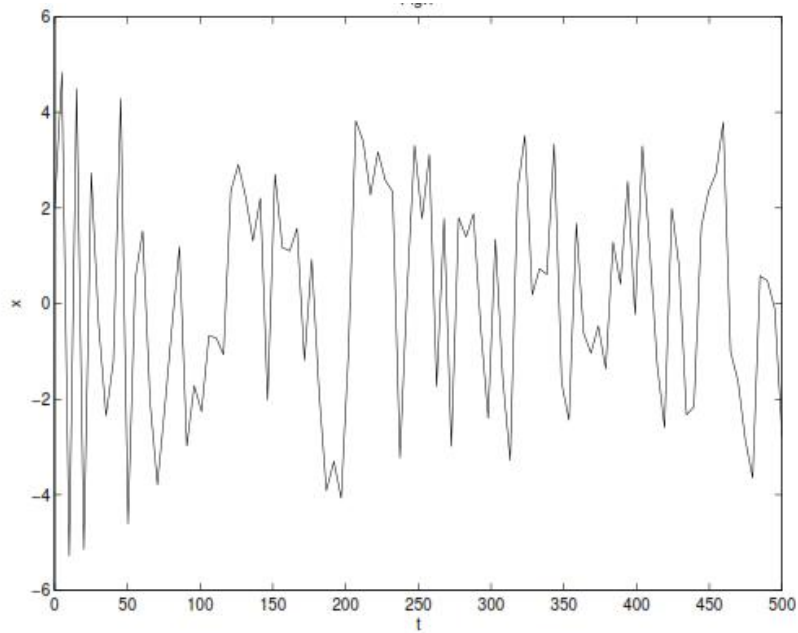


Fig. 7. Chaotic dynamics of Ikeda model x for sinusoidal modulations of the feedback and coupling time delays. Dimensionless units

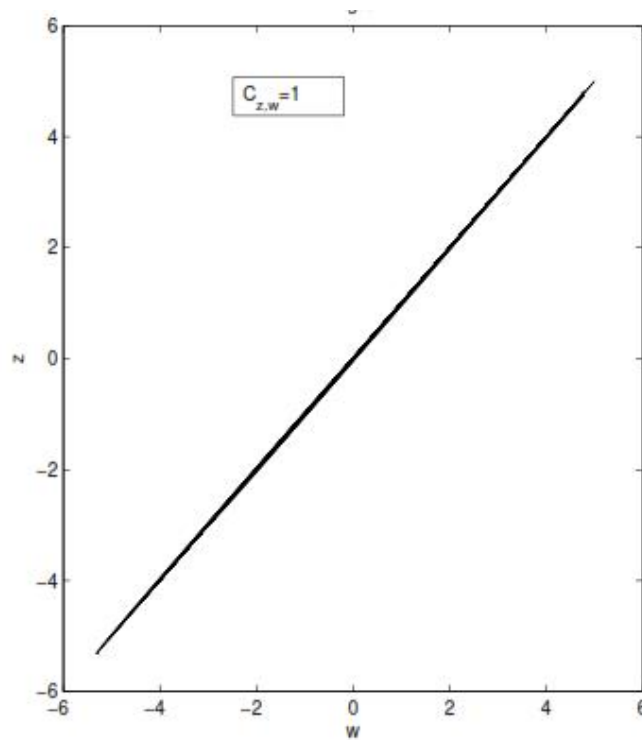


Fig. 8. Synchronization between Ikeda models z and w in case of sinusoidal modulations of the feedback and coupling time delays: z versus w for parameters $\alpha = 3.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau = 5$. Correlation coefficient $C_{z,w} = 1$. Dimensionless units

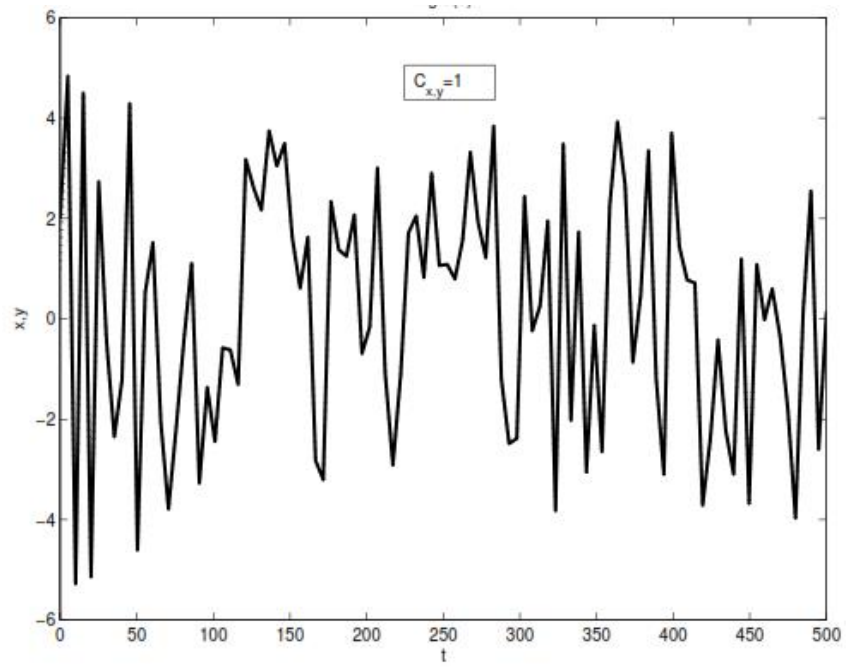


Fig. 8(a). Dynamics of Ikeda models x (solid line) and y (dotted line) for the parameter values as in Fig. 8. Correlation coefficient $C_{x,y} = 1$.

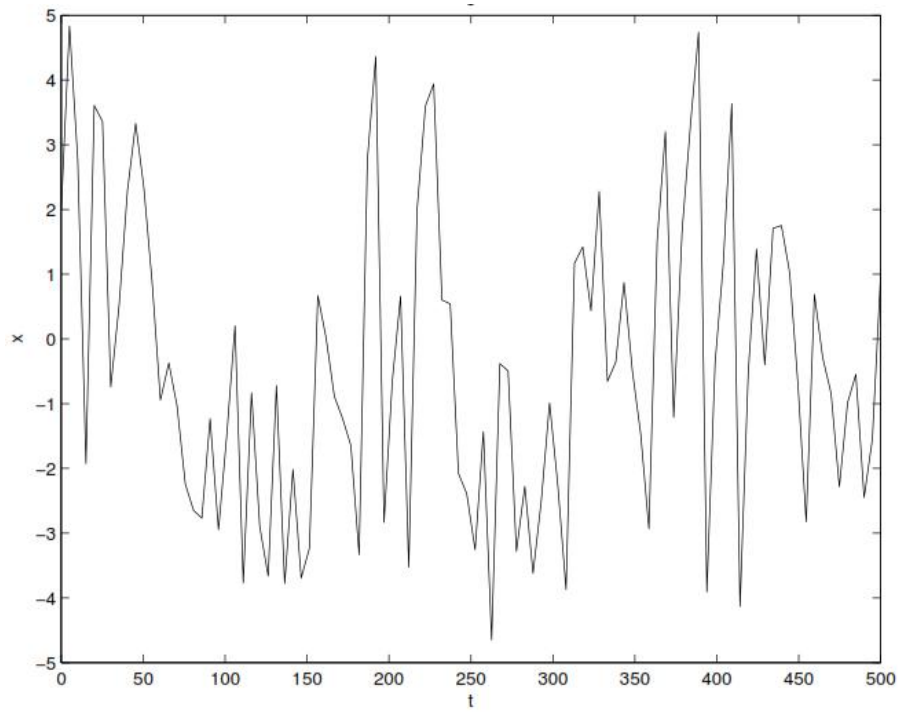


Fig. 9. Chaotic dynamics of Ikeda model x for chaotic modulations $\tau(t) = 5 + 0.8x_1(t)$ of the feedback and coupling time delays. Dimensionless units

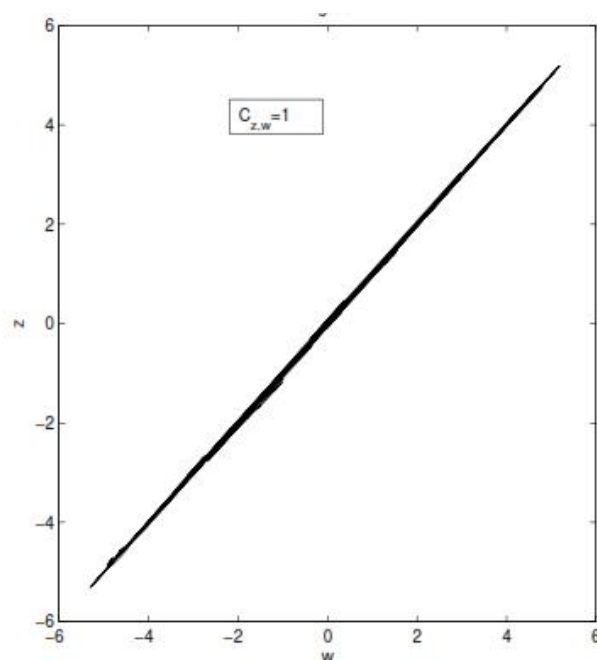


Fig. 10. Synchronization between Ikeda models z and w in case of combined chaotic and sinusoidal modulations of the feedback and coupling time delays: z versus w for parameters

$$\alpha = 3.01, m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m_9 = 8, m_1 = 2m_2 = 16, \tau = 5, \\ \tau(t) = 5 + 0.5x_1(t) \sin(0.1t). \text{ Correlation coefficient } C_{z,w} = 1. \text{ Dimensionless units}$$

It should also be mentioned that chaos synchronisation is not the only phenomenon observed in an ensemble of chaotic systems. Another very interesting phenomenon is the realisation of chimaera states in chaotic systems. In another study [30], the authors have studied dynamical properties of one-dimensional ensembles of identical chaotic oscillators with non-local coupling. The authors have established that such systems can demonstrate the transition from complete chaotic synchronisation to spatiotemporal chaos when the coupling coefficient decreases. This transition is called the coherence – incoherence transition and, for certain networks, is accompanied by the appearance of chimaera states.

Apart from this, breathers and travelling waves can also be observed in some networks [31]. The study of these very interesting phenomena is beyond the scope of this research.

4. CONCLUSIONS

This study reports on all-to-all complete chaos synchronisation in unidirectionally nonlinearly

coupled Ikeda systems. I have considered both constant time delays (feedback and coupling times) and variable time delays cases. In case of constant time delays, analytically the existence and stability conditions for complete chaos synchronisation have been derived. Numerical simulations fully support the analytical findings. As synchronisation is vital in communication systems, these results are of certain importance for information processing purposes. Additionally, the results are useful for obtaining high emission power from such networks. Besides these results testify that driven Ikeda models, although are not coupled directly between themselves, can be synchronised quite robustly by a single driver model even under the conditions of the dithered feedback and coupling time delays. This studied configuration can serve as a *motif* (building block) for much more complex networks.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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