Economies **2014**, *2*, 1-19; doi:10.3390/economies2010001

OPEN ACCESS

economies

ISSN 2227-7099

www.mdpi.com/journal/economies

Article

Financial Markets, Banking and the Design of Monetary Policy: A Stable Baseline Scenario

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Received: 23 October 2013; in revised form: 9 December 2013 / Accepted: 10 December 2013 / Published: 30 December 2013

Abstract: A baseline integration of commercial banks into the disequilibrium framework with behavioral traders of Charpe *et al*. (2011, 2012) is presented. At the core of the analysis is the impact the banking sector exerts on the interaction of real and financial markets. Potentially destabilizing feedback channels in the presence of imperfect macroeconomic portfolio adjustment and heterogeneous expectations are investigated. Given the possible financial market instability, various policy instruments have to be applied in order to guarantee viable dynamics in the highly interconnected macroeconomy. Among those are open market operations reacting to the state-of-confidence in the economy and Tobin-type capital gain taxes. The need for policy intervention is even more striking, as the banking sector is modeled in a rather stability enhancing way, fulfilling its fundamental tasks of term transformation of savings and credit granting without engaging in investment activities itself.

Keywords: monetary business cycles; portfolio choice; credit and banking; (in-)stability; stabilizing policy measures

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1. Introduction

Monetary and fiscal policy measures have been applied in order to avert the financial market collapse of 2008 and counteract the global recession, which has still not been fully overcome. The financial crisis started in the U.S. housing market and was amplified by the bankruptcies of large banks and transmitted, finally, to the real economy. Obviously, it is the interconnectedness of real and financial sectors that makes the working of the whole economy so vulnerable to crashes in one part. We do not claim to capture the recent crisis in its specific lines in our model, but we want to stress that crisis phenomena (even large ones) might not be unique and extremely rare events in capitalist economies, as long as the complex interactions of (macro-) markets remain unfettered.

A basic framework will be delivered, which allows for a unique and attracting steady state despite its high dimension. The model ends up in dimension eight, which implies that the dynamics become easily non-trivial. The strategy to guarantee analytical tractability is to set up the model step-by-step and infer stability properties from the added eigenvalues.

Since the focus of the paper is on the financial side, we model, in addition to basic assets, like bonds and equities, credit relations in detail. Therefore, a baseline integration of commercial banks into the behavioral Keynesian disequilibrium framework of Charpe *et al.* [1,2] will be conducted; noting that we will only use the deterministic part here to set up the framework, though stochastic shocks might be easily reintroduced.[1]

Potentially destabilizing feedback channels of advanced macroeconomies shall be detected and investigated throughout the paper. After the sources of instability are identified, we will design appropriate policy instruments, taking into account the causal structure of the economy. This procedure enables us to display endogenous crisis mechanisms and to highlight conditions (parameter relations) that restrict the occurrence of dynamic instability.

In the model of the paper, we consider, on the one hand, the interaction of asset markets with real economic activity and, on the other hand, the interaction of real activity with the credit channel, here based on commercial banking controlled by the central bank by its money supply policy. Secondary asset markets and real activity are linked via asset price-based demand effects and output-dependent profitability results. Tobin's q will be used here as measure of confidence in the economy, as in Blanchard [4]. This state of confidence matters, then, for consumption and investment decisions, which ultimately drive aggregate demand.

The potential for asset market instability is shown via the coupling of a dynamic Tobinian portfolio approach with the interaction of heterogeneous agents in this market. Brunnermeier [5] reports that the existence of different types of agents in asset markets implying heterogeneous expectation formation is perceived to be one of the main source of bubbles and, thus, instability in financial markets. Therefore, we will use expectation formation schemes of the chartist-fundamentalist variety, as advocated by Menkhoff *et al.* [6] or De Grauwe and Grimaldi [7].

Since we aim to show the potential fragility of the whole economy and not only of certain parts, a bundle of instruments will be needed for an effective cure. Especially Minsky [8,9] developed many ideas of how to stabilize an unstable economy. In our model, Tobin-type capital gain taxation coupled with volatility reducing asset market open market operations of the central bank are capable of making the interaction of goods market results with financial markets a stable one, while additional countercyclical

[1] The paper understands itself as being part of a larger attempt in order to develop a Dynamic Stochastic General Disequilibrium (DSGD) approach to macroeconomics. See, also, Charpe *et al.* [2] and Flaschel *et al.* [3].

money supply rules can make the credit supply a countercyclical one. Taken together, they create the situation of a real financial interaction that allows for attracting steady states despite the high dimensional nature of this interaction, due to the facts that gross substitutability makes financial markets, in principle stable ones (if chartists behavior is not dominating the outcomes on these markets, in particular if supported by countercyclical open market operations on these markets), while the implied credit supply is counteracting booms through credit reductions and busts through credit expansions.

The need for policy intervention is more striking, as the banking sector is modeled in a stability enhancing way, fulfilling its fundamental tasks of term transformation of savings and credit granting without engaging in investment activities itself. This implies that a strict separation between commercial and investment banks is an additional necessary condition for the stable configuration obtained in the end.

2. Tobinian Asset Price Dynamics and the Multiplier

In the Keynesian modeling framework of Charpe *et al.* [1], the interaction of real and financial markets via several potentially destabilizing feedback channels has been investigated. In the following, we start from this framework and integrate a banking sector with the help of credit relationships.

The financial side is described by a Tobin-portfolio structure along the lines of Tobin [10]. The array of financial assets contains equities E, long-term bonds B^l and money represented by short-term bonds B, which are issued by the central bank.

The expectation formation process for capital gains on financial markets is driven by two kinds of agents, namely chartists (showing speculative behavior by making use of a simple adaptive mechanism to forecast price evolution) and fundamentalists (who expect the convergence of capital gains back to their steady-state positions).

At this modeling stage, diverse fiscal and monetary policies are needed in order to stabilize such unstable macroeconomies. One particular policy instrument is the taxation of capital gains. In this paper, the Tobin tax income of the central bank is made explicit contrary to the former approach. The tax revenues are simply transferred into the government sector. When combined with an additionally stabilizing open market policy $\dot{M}$ that buys (sells) the respective assets if the corresponding asset markets are weak (strong), the magnitude of equities and long-term bonds available for private trading becomes endogenous, though overall stocks remain exogenously given. We do not yet consider the issue of new equities by firms or of long-term bonds by the government, *i.e.*, there is no asset accumulation taking place, so far. In this respect, we still ignore the budget equations of firms, the government (and also of the households), due to the simple dynamic multiplier approach we shall be using for the description of the dynamics of the real part of the model.

Complexity on the real side is reduced to a minimum, but the financial structure is extensively modeled. The financial assets are imperfect substitutes, and only a fraction, α, of current stock disequilibria enters the markets for bonds or equities in the form of supply or demand. This is due to the assumption that adjustment costs are implicitly present. Moreover, capital gain expectations are imperfect in the model. This can be justified on empirical grounds: in reality, the gathering of information is quite costly, and information processing capabilities are limited. Attaining perfect foresight (the deterministic correspondence to rational expectations) is out of reach for at least part of the agents acting in financial markets. There might be even a rationale for chartist expectations in the presence of the knowledge of the fundamental positions of the economy. Some agents could perceive that riding the bubble is a favorable strategy, as long as they assume to be smarter than others and exit the market before a potential burst.

In the following, the time derivative of a variable, x, is denoted by $\dot{x}$, the growth rate by $\hat{x}$ and by f_x, the first derivative of a function, $f(\cdot)$, with respect to x. Goods price inflation is not considered, and the corresponding price level normalized to one. Only the equity price, p_e, and the price for long-term bonds, p_b, is assumed to be variable. Y denotes output, $\bar{A}$ autonomous expenditure, r the profit rate, π^e expected capital gains, r_e^e the expected rate of return on equities and r_b^e the expected rate of return on bonds.

The core dynamical system of Charpe *et al.* [1], slightly extended, reads as follows (with $\bar{E}/(pK) = 1$ in Tobin's average *q* for expositional simplicity, *i.e.*, $q = p_e$):

$$\dot{Y} = \beta_y[(a_y - 1)Y + a_q(p_e - p_e^o)E_h + \bar{A}] \tag{1}$$

$$\dot{p}_e\bar{E} = \beta_e\alpha_e(f_e(r_e^e, r_b^e)W_h^n - p_e\bar{E}), \quad r_e^e = \frac{r(Y)}{p_e} + \pi_e^e, 0 < f_e < 1 \tag{2}$$

$$\dot{p}_b\bar{B}^l = \beta_b\alpha_b(f_{b^l}(r_e^e, r_b^e)W_h^n - p_b\bar{B}^l), \quad r_b^e = \frac{1}{p_b} + \pi_b^e, 0 < f_b < 1 \tag{3}$$

with $W_h^n := M + D + p_bB_h^l + p_eE_h$; nominal wealth of households equals money, deposits, the value of long-term bonds and the value of equities.

For further details, see also the accounts shown below. The block of Equations (1)–(3) shows the impact of asset markets on real economic activity, as well as the impact of the profitability[2] $r = \frac{\Pi}{pK}, \frac{r}{p_e} = \frac{\Pi}{p_e\bar{E}}$ of firms on the dynamics of financial markets, where there is also a pronounced self-referencing dynamics at work.

The first law of motion is just the textbook multiplier dynamics, based on Tobin's $q = p_e$, measuring the state of confidence, driving the economy, as far as the feedback from financial markets to the real markets is concerned. The second and third law of motion shows the excess demand pressures, $\alpha_e(\cdot), \alpha_b(\cdot)$, on the respective asset markets, which lead to asset price adjustments with speed β_e, β_b, , but in the end, to no change in the stocks actually held in the private sector. Note also that the shown excess demands for equities and bonds must be balanced (here, implicitly) by a corresponding excess supply of money $M_2 = M + D$ (and *vice versa*). Note further that the stock demand function, f, is characterized by the gross substitute property, *i.e.*, the demand for the respective asset depends positively on its own rate of return, r_x^e, and negatively on the other one's rate of return ($x = e, b$).

3. Stability Propositions

At the heart of our investigation into the interaction of aggregate demand, stock market performance and credit relationships is the question of economic stability. A very convenient way to assess the

[2] Π profits = dividends.

stability characteristics of dynamic systems is to use eigenvalue analysis of the constituting dynamic equations for the state variables. Whenever an eigenvalue of a characteristic polynomial shows a negative real part, convergence can be asserted for the respective law of motion. Associated with deviations from negativity of the real part of an eigenvalue are instability and saddle-point outcomes, whereas even the latter one has to be considered an instability situation from our point of view.[3]

Whether the eigenvalues of low-dimensional systems are positive or negative can be evaluated by means of Routh–Hurwitz conditions, which make use of the Jacobian matrices. The Jacobian matrices contain the partial derivatives evaluated at the steady state. From these, one has to determine the trace and determinant in order to check the fulfillment of the theorems.[4]

All proofs are given directly in the text, except those establishing the core system of real-financial interactions without a banking sector, which can be found in the Appendix.

We get from the above the following proposition for the stability of the asset markets when capital gain expectations are static.

Proposition 1: Stable Financial Markets Interaction

Assume that capital gain expectations are static. Then, the dynamics:

$$\dot{q} = \beta_e \alpha_e (f_e(r_e^e, r_b^e) W_h^n - q), \quad q = p_e \bar{E}, \alpha_e \in (0,1) \tag{4}$$

$$\dot{b} = \beta_b \alpha_b (f_{b^l}(r_e^e, r_b^e) W_h^n - b), \quad b = p_b \bar{B}^l, \alpha_b \in (0,1) \tag{5}$$

converges to the current asset market equilibrium for all adjustment speeds of asset prices p_e, p_b.

The proof of Proposition 1 is presented in the Appendix and is based on Charpe *et al.* [1]. Since the dynamic multiplier of the real side is stable on its own, we can state that in sum, the real and the financial markets, when considered in isolation (and with sufficiently tranquil capital gain expectations) are both stable. The next step, therefore, is to investigate what happens when they are interacting as a full 3D dynamical system.

In the case of static chartist capital gain expectations, we obtain the following proposition:

Proposition 2: Stable 3D Real-Financial Markets Interaction

Assume that the parameter, β_y*, is sufficiently large and the parameter* β_e *sufficiently small. Assume, moreover, that the parameter,* a_y*, is sufficiently close to one (but smaller than one). Then, the dynamics:*

$$\dot{Y} = \beta_y[(a_y - 1)Y + a_q(q - q^o) + \bar{A}] \tag{6}$$

$$\dot{q} = \beta_e \alpha_e (f_e(r_e^e, r_b^e) W_h^n - q) \tag{7}$$

$$\dot{b} = \beta_b \alpha_b (f_{b^l}(r_e^e, r_b^e) W_h^n - b) \tag{8}$$

is locally asymptotically stable around its steady-state position.

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3 A very extensive discussion on this point can be found in Chiarella *et al.* [11], as well as in Chiarella, Flaschel and Semmler [12].

4 Compare e.g., Tu [13]

This proposition and its proof (modified from Charpe *et al.* [1]; see the Appendix) show, however, that the coupling of two stable, but partial, processes need not provide a stable interaction of the two partial processes. The stability proposition is restricted with respect to several parameters and their possible values. In general, stability cannot be ascertained for the whole parameter space. Moreover, until now, the working of financial markets has been relatively tranquil compared to the stage when capital gains will be considered. These capital gain expectations will be a serious source of instability if the weight of chartists in the average market expectations is high.

Contrasting these propositions with the case of neoclassical perfectness with regard to substitution and expectations delivers, basically, the Blanchard [4] model and its stability implications. Perfectness demands $\beta_e, \beta_b = \infty$ and $\alpha_e, \alpha_b = 1$, which means perfect substitution of assets and myopic perfect foresight of the capital gains evolution. The 3D core system then collapses into a two-dimensional one in Y and q. The system is unstable, but exhibits a saddle path. The neoclassical treatment of such a stability situation requires the usage of the jump-variable technique of Sargent and Wallace [14]. It is assumed that the economy always jumps to the converging trajectory. The possibility of an unstable economy is ruled out by assumption. An in-depth demonstration and critique of this technique can found in Chiarella *et al.* [11].

4. Commercial Banking and Central Bank Behavior

This model of the private sector of the economy is now augmented by a detailed description of the banking sector and the policy actions of the central bank. In particular, the central bank's income of the required Tobin capital gain taxation is dealt with in detail, and the credit channel of the economy will be introduced explicitly. First, the stock and flow accounts of the central bank (Tables 1 and 2) are presented in order to capture its full activities.

Table 1. The balance sheet of the central bank.

Assets	***Liabilities***
Treasury bonds (perpetuities): $p_b B_c^l \quad [B_c^l = \bar{B}^l - B_h^l]$	High powered money: $\mathcal{M} = M + R$
Equities (from firms): $p_e E_c \quad [E_c = \bar{E} - E_h]$	CB: net worth

Table 2. The monetary policy (flows) of the central bank.

Uses	***Resources***
	OMP: $\dot{\mathcal{M}} = c_{p_e}(p_e^o - p_e)E_h + c_{p_b}(p_b^o - p_b)B_h^l$
Equity demand $p_e\,\dot{E}_c^[= c_{p_e}(p_e^o - p_e)E_h$	Tobin capital gains taxes $\tau_e\,\dot{p}_e\,E_h$
Bond demand $p_b\dot{B}_c^l = c_{p_b}(p_b^o - p_b)B_h^l$	Tobin capital gains taxes $\tau_b\dot{p}_b B_h^l$
CB surplus ($\rightarrow$government budget constraint (GBR)): $rE_c + B_c^l + \tau_e\dot{p}_e E_h + \tau_b\dot{p}_b B_h^l$	Dividends and interest $rE_c + B_c^l$

The balance sheet simply states that the central bank can hold treasury bonds issued by the government or equities issued by firms. Since the stock of these assets is assumed to be fixed for the time being, the difference between total stocks and central bank's holdings must be private holdings.

The flow account[5] shows on the right-hand side the resources that accrue to the central bank, namely dividends on its stock holdings (r, the rate of profit of firms) and interest on its long-term bond holdings, the taxes that are obtained by capital gain taxation and the changes in its inventory of equity and long-term bonds through its open market operations $\dot{\mathcal{M}}$. These changes are the uses (if positive) of the issue of new money, $\dot{M}$, and are reported on the left-hand side of its income account again.[6]

Against this background, we have to consider now the two laws of motion for the households' holdings of equities and long-term bonds, *i.e.*, of the endogenous variables, E_h, B_h^l, which are given by:[7]

$$p_e\dot{E}_h = -c_{p_e}(p_e^o - p_e)E_h, \quad E_h = \bar{E} - E_c \tag{9}$$

$$p_b\dot{B}_h^l = -c_{p_b}(p_b^o - p_b)B_h^l, \quad B_h^l = \bar{B}^l - B_c^l \tag{10}$$

They imply, as laws of motion for private equity and long-term bond holding:

$$\hat{E}_h = -c_{p_e}(p_e^o/p_e - 1) \tag{11}$$

$$\hat{B}_h^l = -c_{p_b}(p_b^o/p_b - 1) \tag{12}$$

We only observe here that these two laws of motion do not endanger the stability of the 3D baseline structure, at least when operated in a sufficiently cautious way, since the implied 4D system exhibits a positive determinant and the implied 5D dynamics, when the second law is added, a negative one, so that parameter changes with respect to c_{p_e} and c_{p_b}, from zero to small values of them, will add negative eigenvalues to the already existing three eigenvalues with their negative real parts.

Next, we introduce commercial banks into the model and enlarge therewith the assumed financial structure of money, M, long-term bonds B^l and equities E by deposits D and loans L. Commercial banks are here conceived as firms that hold saving deposits D from households and transform them into credit to firms, based on reserve requirements $R = \delta_r D$. Banks are borrowing short and lending long, which means that they provide for the term transformation of savings.[8] This is their only proper function, *i.e.*, they, in particular, do not trade on financial markets in an active way, but only passively adjust their interest rate on deposits, i_d, such that the flow account shown below is a balanced one with respect to the new loan supply, $\dot{L}$, they intend to provide.[9]

We assume, as in the multiplier approach to money holdings $M_2 = M + D$, that households are hoarding (in secured deposit boxes) as "cash" $M = \delta_h D$ a certain amount of money, M of the central

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5 $rE_c = \Pi\frac{E_c}{E}$; all dividends are paid out as profits.

6 We assume that the central bank surplus is transferred into the government sector for expositional simplicity, *i.e.*, it affects the government budget constraint (GBR).

7 Note that we do not yet integrate the new issue of equities by firms and of new bonds issued by the government.

8 This is the basic and fundamental task attributed to banks. On this, see, for example, Gorton and Winton [15] or Freixas and Rochet [16]. The latter, moreover, deliver rigorous microeconomic justifications for the existence of financial intermediaries.

9 We abstract from labor input and equipment in the banking sector here.

bank, assumed to be proportional to the deposit, D, they have accumulated, *i.e.*, they do not deposit all their money holdings, M_2, into the credit circuit operated by commercial banks. The supply of money by the central bank is then characterized by:

$$\mathcal{M} = M + R = (\delta_h + \delta_r)D$$

as the relationship between the money supplied by the central bank and the deposits held in the household sector. The total amount of money, M_2, held in the household sector, moreover, by definition is:

$$M_2 = D + M = (1 + \delta_h)D$$

This gives, as usual, the money multiplier formula:

$$M_2 = M + D = \frac{1 + \delta_h}{\delta_h + \delta_r}\mathcal{M} = \alpha_m \mathcal{M}$$

between the money supply concept, M_2, and the money supply, $M+R$, issued by the central bank. Based on this multiplier formula, we can now redefine the nominal wealth of households into the four assets, $M + D, B_h^l, E_h$, that households now possess by:

$$W_h^n := \alpha_m \mathcal{M} + p_b B_h^l + p_e E_h$$

We assume here that the interest rate on savings deposits, i_d, only concerns the allocation of M_2 into cash and savings deposits and, thus, the cash management process of the households $M = \delta_h(i_d)D$, while the allocation of financial assets between liquid assets M_2 and bonds and equities is driven by the rates of return on the risky assets solely. Moreover, we could also assume that the reserve rate is a given magnitude, controlled (fixed) by the central bank, which, thereby, then influences the supply of new credit, a scenario that does impact the model in its present formulation.

Commercial banks adjust the loan rate, i_l, in order to bring credit supply in line with credit demand. The credit multiplier can then be easily calculated, providing the expression:[10]

$$L = (1 - \delta_r)D = \frac{1 - \delta_r}{\delta_h + \delta_r}\mathcal{M} = \alpha_l \mathcal{M}$$

Based on the total supply of high powered money: $\mathcal{M} = M + R = \delta_h D + \delta_r D$ and the shown demand for it, as well as the subsequent derivation of the multiplier formulae for money and credit, the banking sector exhibits the following balance sheet and flow account(Tables 3 and 4).[11]

Table 3. The balance sheet of the commercial banks (private ownership).

Assets	***Liabilities***
Loans $L = \frac{1-\delta_r}{\delta_r+\delta_h}\mathcal{M}$	Households' saving deposits $D = (1 - \delta_h)\frac{1+\delta_h}{\delta_r+\delta_h}\mathcal{M}$
Reserves $R = \delta_r D$	

[10] When the deposits that flow back from the granted loans via the circuit of money were considered to become new loans again, the loan multiplier would alter to $L = \frac{1-\delta_r}{\delta_r}D$. The magnitude of the multiplier would change, but, of course, not its qualitative direction.

[11] It should be noted that we assume banks to be privately owned for reasons of expositional simplicity.

Table 4. The flow account of the commercial banks (private ownership).

Uses	***Resources***
Interest rate payments $i_d D, i_d = \frac{i_l^o}{1+\mu}$	Loan rate income $i_l^o L$; i_l^o, the market clearing loan rate
Reserve adjustment $\delta_r \dot{D}$	New deposits $\dot{D}$
New loan supply $\dot{L}$	

The maturity transformation function of banks is reflected in the balance sheet. Banks collect private savings, which are checking deposits, and transform them into loans that possess a longer maturity horizon. The creation of loans is constrained by the reserve obligation in the form of a certain fraction of deposits, $\delta_r D$, not to be granted as loans.[12]

The relationship between high powered money, $\mathcal{M}$, and the volume of loans, L, gives rise to the following law of motion for the loans of commercial banks to firms:

$$\dot{L} = \alpha_l \dot{\mathcal{M}} = \alpha_l [c_{p_e}(p_e^o - p_e)E_h + c_{p_b}(p_b^o - p_b)B_h^l] \tag{13}$$

$$\hat{E}_h = -c_{p_e}(p_e^o/p_e - 1) \tag{14}$$

$$\hat{B}_h^l = -c_{p_b}(p_b^o/p_b - 1) \tag{15}$$

This shows that this specific integration of commercial banks into our model adds one further law of motion (for loans) to the ones already considered.

Moreover, we postulate now as a law of motion for the output of firms on the basis of an extended aggregate demand schedule (now based on the excess debt level, $L - L^o$, transmitted through firms' investment demand schedule, as well as the state of confidence measure q):

$$\dot{Y} = \beta_y[(a_y - 1)Y + a_q(p_e - p_e^o)E_h - a_l(L - L^o) + \bar{A}]$$

if we assume that the private sector considers the steady-state loan volume as normal and deviations from it as impacting investment and the economy in a negative way. This can be justified by the assumption that firms take into account their stock of debt in their investment decisions. High leverage levels expose firms to the risk of insolvency; low leverage states induce expansions of investment projects. The incorporation of the credit channel makes the dynamical system now an eight-dimensional one.

5. Credit and Credit-Dependent Multiplier Dynamics

We already described the banking activity as a transformational one: Private savings are channeled into loans and enable investment activities. Banks serve as financial intermediaries that allow for term conversion. The link between the real side and the credit channel implies the interaction of the output multiplier with the loan multiplier and should be investigated first in isolation from the overall dynamics.

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[12] The framework would allow for a separate treatment of time and checkable deposits. The distinction between time and checkable deposits plays an important role in Chiarella *et al.* [17] and Flaschel *et al.* [18] when the narrow banking idea of Irving Fisher is discussed in such a disequilibrium modeling approach.

Adding the debt dynamics to the financial markets gives, by means of the money supply rule of the central bank, the two further laws of motion:

$$\dot{L} = \alpha_l[c_{p_e}(p_e^o - p_e)E_h + c_{p_b}(p_b^o - p_b)B_h^l]$$

$$\dot{Y} = \beta_y[(a_y - 1)Y + a_q(p_e - p_e^o)E_h - a_l(L - L^o) + \bar{A}]$$

Concerning the first law of motion, we have zero root hysteresis in the evolution of the state variable, L, while the output dynamic, $\dot{Y}$, adds a stable dynamic multiplier process to the financial sector of the economy. Moreover, a monetary policy, which increases credit in busts and decreases its volume in the boom, should also contribute to the stability of the overall real-financial market interaction.

We now augment the dynamics of the money supply, however, as follows:

$$\dot{\mathcal{M}} = c_{p_e}(p_e^o - p_e)E_h + c_{p_b}(p_b^o - p_b)B_h^l + \gamma_y Y - \gamma_m \mathcal{M}$$

We here assume that the time rate of change of the money supply (and, thus, credit) is also influenced by the state of the business cycle in a countercyclical way and that there is also a negative feedback of the level of money supply on its current rate of change. We assume that this change in money supply is deducted from the central bank gains that are distributed to the government.

The output-debt subdynamics of the model then read:

$$\dot{L} = \alpha_l[c_{p_e}(p_e^o - p_e)E_h + c_{p_b}(p_b^o - p_b)B_h^l + \gamma_y Y - \gamma_m L/\alpha_l]$$

$$\dot{Y} = \beta_y[(a_y - 1)Y + a_q(p_e - p_e^o)E_h - a_l(L - L^o) + \bar{A}]$$

and are, in themselves, asymptotically stable if the propensity to spend is less than one (as it is usually assumed):

$$J(L, Y) \quad = \quad \begin{pmatrix} - & + \\ - & - \end{pmatrix}$$

The trace is negative, and the determinant is positive, which implies two negative eigenvalues according to Routh–Hurwitz conditions.

6. Financial Markets, Accelerating Capital Gain Expectations and Tobin-Type Taxes

The picture of the fully interacting eight-dimensional system is still not complete. For the moment, we concentrate on the dynamics of financial markets and the impact of monetary policy on these dynamics, by keeping the real sector and credit fixed at their steady-state values. The output and loan dynamics is thereby ignored in the following, as is the dynamics of asset accumulation through investment and the corresponding issue of equities, as well as the government budget constraint and the corresponding issue of new government bonds.

We thus first study the financial markets in isolation. We add to them now expectations schemes, as far as static expected capital gains were employed before. By endogenizing capital gain expectations, we distinguish between fundamentalists f and chartists c and assume for the former that they expect capital gains to converge back with speeds $\beta_{\pi_{ef}}, \beta_{\pi_{bf}}$ to their steady-state position, which is zero. Chartists, by contrast (for analytical simplicity), make use of a simple adaptive mechanism to forecast the evolution of capital gains, $\dot{\pi}_e^e$, in the equity market and $\dot{\pi}_b^e$ in the market for long-term bonds. Market expectations π^e are then an average of fundamentalist and chartist expectations.

Justification for this scheme is two-fold: Many empirical studies argue in favor of this kind of expectation mechanisms in order to explain agents' behavior on financial or FXmarkets and the ability to be insightful despite its relatively simple structure. De Grauwe and Grimaldi [7] employ this kind of scheme to characterize the behavior of agents on the foreign exchange market, and Brunnermeier [5] shows how bubbles can evolve in a market when this agent constellation is underlying.[13]

We stress here that these simple expectation formation mechanisms are chosen to make the dynamics analytically tractable. They can, of course, be replaced by much more refined forward- and backward-looking expectation rules when the model is treated numerically. However, we do not expect that this changes the results in a significant way if these learning mechanisms are built in the spirit of the ones we introduce and employ below.

The incorporation of imperfect expectation schemes gives, finally, a rise to the following set of equations:

$$\begin{aligned}
\dot{p}_e E_h &= \beta_e \alpha_e (f_e(r_e^e, r_b^e) W_h^n - p_e E_h), \quad r_e^e = \frac{r(Y)}{p_e} + \pi_e^e && (16) \\
\dot{p}_b B_h^l &= \beta_b \alpha_b (f_b(r_e^e, r_b^e) W_h^n - p_b B_h^l), \quad r_b^e = \frac{1}{p_b} + \pi_b^e && (17) \\
\hat{E}_h &= -c_{p_e}(p_e^o / p_e - 1) && (18) \\
\hat{B}_h^l &= -c_{p_b}(p_b^o / p_b - 1) && (19) \\
\dot{\pi}_e^e &= \beta_{\pi_e^{ec}}((1 - \tau_e)\hat{p}_e - \pi_e^e) + \beta_{\pi_e^{ef}}(0 - \pi_e^e) && (20) \\
\dot{\pi}_b^e &= \beta_{\pi_b^{ec}}((1 - \tau_b)\hat{p}_b - \pi_b^e) + \beta_{\pi_b^{ef}}(0 - \pi_b^e) && (21)
\end{aligned}$$

with $W_h^n = \alpha_m \mathcal{M} + p_b B_h^l + p_e E_h$. Note that we, in addition, have the law of motion:

$$\dot{\mathcal{M}} = c_{p_e}(p_e^o - p_e) E_h + c_{p_b}(p_b^o - p_b) B_h^l$$

which feeds back into the rest of the dynamics through the definition of private wealth. This law of motion, when added to the above dynamics, does, however, not alter them very much, since it gives rise to a zero root and, thus, to zero root hysteresis in the money supply, $\mathcal{M}$, solely. Nominal money supply and its steady-state value, and, thus, private wealth, is, therefore, a path-dependent state variable in this version of the model.

The remaining steady-state values of the above dynamics are, of course, simply given by:

$$p_e = p_e^o, p_b = p_b^o, \pi_e^{eo} = 0, \pi_b^{eo} = 0$$

For the financial markets subdynamics, the following propositions are obtained:

[13] See Charpe *et al.* [2] for endogenizing the population weights of each type of agent and also Proaño [19] for the incorporation of heterogeneous expectations in a two-country model along the lines of the disequilibrium approach to macroeconomics pursued also in this paper. For empirical evidence on the chartist-fundamentalist framework in explaining expectational heterogeneity, see Menkhoff *et al.* [6].

Proposition 3: Gross substitutes, stabilizing expectations and absence of monetary policy

Assume that output Y is fixed at its steady-state value. Then, the 4D dynamics, (16), (17), (20), (21), of asset prices is asymptotically stable around its steady-state position if capital gain expectations are dominated by fundamentalists to a sufficient degree (which can be enforced by choosing the Tobin tax parameters as sufficiently high).

Proof: The proposition is a consequence of the inherently stabilizing Tobinian gross substitute assumption for equities and long-term bonds; see Flaschel *et al.* [18]. □

This shows that the financial core of the model can work in a proper way with respect to local stability under conditions that favor fundamentalist behavior in the asset markets, a prerequisite that cannot be easily regarded as being given *per se* without policy intervention, as it is by no means clear that fundamentalist expectations will dominate chartistic ones. At least the gross substitution characteristic facilitates the requirements for convergence, since negative dependence of demand for one asset on the rate of return of the other one excludes explosiveness from this source of interaction.

Proposition 4: Gross substitutes, static expectations and monetary policy

Assume that output Y is fixed at its steady-state value. Then, the 4D dynamics, (16)–(19), of asset prices, with one or two policy rules switched on, is asymptotically stable around the steady-state position for all choices of the policy parameters in laws of motion (20) and (21).

Proof: Consider the countercyclical equity policy rule of the central bank. Then, it is easy to show that the resulting 3D Jacobian of the considered subdynamics fulfills that $a_1 = -\text{trace } J$ is positive; the positive 2D upper principal minor is increased by $-J_{13}J_{31}$, and $a_3 = -\det J$ is positive and equal to $J_{22}J_{13}J_{31}$, so that all Routh–Hurwitz polynomial coefficients are positive. The expression, $a_1a_2 - a_3$, finally is positive, since $a_3 = J_{11}J_{13}J_{31}$ is contained in a_1a_2. The same applies to the bond-oriented monetary policy.

The combination of the two policies leads to a positive 4D determinant (obtained by several appropriate row operations), so that the stability results are preserved for small positive policy parameters, since the determinant is the product of the four eigenvalues and since three of them already have negative real parts. However, the final Routh–Hurwitz condition:

$$a_1a_2a_3 - a_1^2a_4 - a_3^2 = (a_1a_2 - a_3)a_3 - a_1^2a_4$$

leads to lengthy expressions, where the dominance of the positive terms over the negative terms is not so obvious, as in the discussed 3D case. □

Therefore, Proposition 4 provides the foundation for the impact of monetary policy. The potentially beneficial effects of the countercyclical policy rules of the central bank for the real sector of the economy do not lead to instability in its financial part. The interaction of asset demand and asset supply works smoothly when regarded in isolation.

Proposition 5: Tobin type transactions costs or capital gains taxation

The full 6D dynamics is asymptotically stable around the steady-state position of its state variables if the Tobin-type capital gain taxation parameters are chosen as sufficiently high or if Tobin-type transaction costs on financial markets make the parameters, α_e, α_b, sufficiently small.

Proof: In the limit case where zero is enforced for transactions or capital gains, we get:

$$\dot{\pi}_e^e = \beta_{\pi_e^e}(0 - \pi_e^e) \tag{22}$$

$$\dot{\pi}_b^e = \beta_{\pi_b^e}(0 - \pi_b^e) \tag{23}$$

as further laws of motion, which makes the 6D dynamics, in a trivial way, locally asymptotically. □

This proposition thus demands, as a lot of literature starting with Tobin [20], Tobin-type taxation rules concerning financial market transactions or sufficiently high taxes on capital gains.[14]

Inserting the various equations of the model into each other as far as the structure of financial markets is concerned gives, however:

$$\dot{\pi}_e^e = \beta_{\pi_e^e}((1-\tau_e)\beta_e[\alpha_e(f_e(\frac{1}{p_e} + \pi_e^e, \frac{r(Y)}{p_b} + \pi_b^e)[\alpha_m \mathcal{M} + p_b B_h^l + p_e E_h] - p_e E_h)] - \pi_e^e) \tag{24}$$

$$\dot{\pi}_b^e = \beta_{\pi_b^e}((1-\tau_b)\beta_b[\alpha_b(f_b(\frac{1}{p_e} + \pi_e^e, \frac{1}{p_b} + \pi_b^e)[\alpha_m \mathcal{M} + p_b B_h^l + p_e E_h] - p_b B_h^l)] - \pi_b^e) \tag{25}$$

These equations show that the trace of the Jacobian can be made positive (by means of its fifth and sixth component) if, for example, the parameter combinations:

$$\beta_{\pi_e^e}(1-\tau_e)\beta_e \alpha_e f_{e\pi_e^e} > 1, \beta_{\pi_b^e}, (1-\tau_b)\beta_b \alpha_b f_{b\pi_b^e} > 1$$

are made sufficiently large through an appropriate choice of the β_{π^e}-adjustment speed variables.

These speeds of adjustment must be dampened in any case through monetary policy measures in order to avoid the emergence of asset market bubbles that can endanger the stability of the economy. The trace of *J* becomes positive only after the system has already lost its stability by way of a Hopf-bifurcation. The trace = 0 condition, therefore, supplies only an upper bound for the parameter region, where the system can be expected to be stable.

7. Financial Markets, Credit and Output Dynamics: A Stable Baseline Scenario

The full system describes an economy being complete with regard to all basic financial instruments from a macro perspective. Therefore, the full interaction of the financial side (including credit and the capital gains acceleration process) with the real side (including a Tobinian investment accelerator) can be investigated now.

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[14] Westerhoff [21], as well as Dieci and Westerhoff [22] show the volatility-reducing effect of Tobin taxes also in a heterogeneous trader's framework.

J(8,8) =

$$\begin{pmatrix}
\delta\dot{p}_e E_h/\delta p_e & \delta\dot{p}_e E_h/\delta p_b & \delta\dot{p}_e E_h/\delta E_h & \delta\dot{p}_e E_h/\delta B_l & \delta\dot{p}_e E_h/\delta p_e & \delta\dot{p}_e E_h/\delta p_e & \delta\dot{p}_e E_h/\delta L & \delta\dot{p}_e E_h/\delta Y \\
\delta\dot{p}_b B_h^l/\delta p_e & \delta\dot{p}_b B_h^l/\delta p_b & \delta\dot{p}_b B_h^l/\delta E_h & \delta\dot{p}_b B_h^l/\delta B_l & \delta\dot{p}_b B_h^l/\delta & \delta\dot{p}_b B_h^l/\delta & \delta\dot{p}_b B_h^l/\delta L & \delta\hat{p}_b B_h^l/\delta Y \\
\delta\hat{E}_h/\delta p_e & \delta\hat{E}_h/\delta p_b & \delta\hat{E}_h/\delta E_h & \delta\hat{E}_h/\delta B_l & \delta\hat{E}_h/\delta & \delta\hat{E}_h/\delta & \delta\hat{E}_h/\delta L & \delta\hat{E}_h/\delta Y \\
\delta\hat{B}_l/\delta p_e & \delta\hat{B}_l/\delta P_b & \delta\hat{B}_l/\delta E_h & \delta\hat{B}_l/\delta B_l & \delta\hat{B}_l/\delta & \delta\hat{B}_l/\delta & \delta\hat{B}_l/\delta L & \delta\hat{B}_l/\delta Y \\
\delta\dot{\pi}_e^e/\delta p_e & \delta\dot{\pi}_e^e/\delta p_b & \delta\dot{\pi}_e^e/\delta E_h & \delta\dot{\pi}_e^e/\delta B_l & \delta\dot{\pi}_e^e/\delta & \delta\dot{\pi}_e^e/\delta & \delta\dot{\pi}_e^e/\delta L & \delta\dot{\pi}_e^e/\delta Y \\
\delta\dot{\pi}_b^e/\delta p_e & \delta\dot{\pi}_b^e/\delta p_b & \delta\dot{\pi}_b^e/\delta E_h & \delta\dot{\pi}_b^e/\delta B_l & \delta\dot{\pi}_b^e/\delta & \delta\dot{\pi}_b^e/\delta & \delta\dot{\pi}_b^e/\delta L & \delta\dot{\pi}_b^e/\delta Y \\
\delta\dot{L}/\delta p_e & \delta\dot{L}/\delta p_b & \delta\dot{L}/\delta E_h & \delta\dot{L}/\delta B_l & \delta\dot{L}/\delta & \delta\dot{L}/\delta & \delta\dot{L}/\delta L & \delta\dot{L}/\delta Y \\
\delta\dot{Y}/\delta p_e & \delta\dot{Y}/\delta p_b & \delta\dot{Y}/\delta E_h & \delta\dot{Y}/B_l\delta & \delta\dot{Y}/\delta & \delta\dot{Y}/\delta & \delta\dot{Y}/\delta L & \delta\dot{Y}/\delta Y
\end{pmatrix}$$

For the full dynamics with L as the seventh and Y as the eighth state variable, we have as signs of the Jacobian at the steady state:

$$J(8,8) \quad = \quad \begin{pmatrix}
- & + & - & 0 & + & - & 0 & + \\
+ & - & 0 & - & - & + & 0 & 0 \\
+ & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & + & 0 & 0 & 0 & 0 & 0 & 0 \\
- & + & 0 & 0 & \pm & 0 & 0 & + \\
+ & - & 0 & 0 & 0 & \pm & 0 & 0 \\
- & - & 0 & 0 & 0 & 0 & - & + \\
+ & 0 & 0 & 0 & 0 & 0 & - & -
\end{pmatrix}$$

By the usage of appropriate row operations, one can remove the influence of the first six state variables from the last two rows, *i.e.*, in the full 8D case, the 2D subdeterminant can be isolated in its row representation from the first six state variables and the corresponding subdeterminant. The 8D determinant is, therefore, positive.

Positive feedback loops, leading to instability via the $a_2 > 0$ Routh–Hurwitz stability condition, are $J_{18}J_{81}$, $J_{55}J_{88}$, $J_{66}J_{88}$. They appear to imply instability if β_y is made sufficiently large (a_y is sufficiently close to one). The system is therefore generally not stable, even if the 6D and the 2D systems are assumed as stable when isolated from each other. This is basically due to the fact that economic activity depends positively on Tobin's q and Tobin's q positively on economic activity via the rate of profit. The dynamics is, however, 8D stable if β_y is chosen as sufficiently small, since the zero eigenvalue must become negative if β_y is made positive (the full determinant is positive).

There is, therefore, only a limited reason that the full 8D dynamics can be made asymptotically stable through the policy instruments of the central bank (Tobin taxes and open market operations and countercyclical money supply actions) if the positive feedbacks, $J_{18}J_{81}$, $J_{55}J_{88}$ and $J_{66}J_{88}$, become sufficiently large.

If asset markets are booming, the money supply is decreased by the selling of financial assets through the central bank (and *vice versa*). This, in particular, should moderate the volatility in the financial markets and, thus, contribute to the overall stability of the interaction of the real with the financial sector.

Assume that the 6D system characterizing the financial markets is stable and that capital gain taxes are sufficiently large for this purpose, as well as for a reduction of the entry, J_{58}. Assume, finally, that dividends are taxed, such that $r'(Y)$ becomes sufficiently small. In the limit, the sign structure of the Jacobian is then characterized by:

$$J(8,8) \quad = \quad \begin{pmatrix} - & + & - & 0 & + & - & 0 & 0 \\ + & - & 0 & - & - & + & 0 & 0 \\ + & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & + & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & - & 0 & 0 \\ - & - & 0 & 0 & 0 & 0 & - & + \\ + & 0 & 0 & 0 & 0 & 0 & - & - \end{pmatrix}$$

which implies the stability of the full dynamics, since $|\lambda I - J|$ can then be decomposed into the financial and the real part of the model. The eigenvalue structure of the full 8D system is identical with the eigenvalue structure that results when the 2D subsystem and 6D subsystem are regarded consecutively. The entry, $J_{18}J_{81}$, is then no longer causing problems (also, if chosen as sufficiently small), because it only appears in the determinant and not in a subdeterminant.

Without a limitation of the results obtained beforehand, a credit expansion effect could be added to the aggregate demand schedule. Obviously, $\dot{L}$ should feed back into the output dynamics. The expression for the dynamic multiplier would alter to:

$$\dot{Y} = \beta_y[(a_y - 1)Y + a_q(p_e - p_e^o)E_h - a_l(L - L^o) + \dot{L} + \bar{A}]$$

since loan increases immediately generate the same amount of aggregate demand.

Regarded in isolation, this would add another source of instability, but embedded into the full dynamics, its destabilizing potential vanishes. The slightly modified Jacobian looks like:

$$J(8,8) \quad = \quad \begin{pmatrix} - & + & - & 0 & + & - & 0 & 0 \\ + & - & 0 & - & - & + & 0 & 0 \\ + & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & + & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & - & 0 & 0 \\ - & - & 0 & 0 & 0 & 0 & - & + \\ \pm & - & 0 & 0 & 0 & 0 & - & \pm \end{pmatrix}$$

and shows, now, an ambiguity concerning the sign in J_{81} and J_{88}, as well as a negative entry in J_{82}. In the limit case, J_{81} cancels out again as J_{82} does. The positive influence of output on its rate of change is caused by the monetary policy that cares about the state of the business cycle via $\gamma_y Y$. For a stable working of the whole economy, it is just necessary to conduct this policy carefully enough in order not to fully counteract the stable dynamic multiplier.

8. Conclusions and Outlook

A framework with all basic financial markets (including credit relationships) from a macroeconomic perspective was presented. Sources of instability were highlighted and remedies to overcome these fragilities proposed. Financial market-oriented open market policies, augmented by a procyclical term and Tobin-type capital gain taxation do the job of stabilizing the economy characterized by a Tobinian financial structure and heterogeneous expectation formation. Summing up, we therefore get that the financial markets must be regulated and also handled by open market policy with care, in order to ensure the stability of the real-financial market interaction in the considered economy. Of course, these conditions are only sufficient ones and, thus, not necessary ones, in order to have such stability assertions. The provided stable baseline model might serve as a reference framework and point of departure for the discussion of further topics of banking and asset markets in a macroeconomic context.

Acknowledgments

We would like to thank three anonymous referees for their constructive and helpful comments.

Conflicts of Interest

The authors declare no conflicts of interest.

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A. Appendix

Proof of Proposition 1:

The matrix of partial derivatives of the two considered laws of motion is given by:

$$J = \begin{pmatrix} \beta_e\alpha_e[f_{e1}(\cdot)(-r/q^2)W_h^n + (f_e-1)\bar{E}] & \beta_e\alpha_e[f_{e2}(\cdot)(-1/p_b^2)W_h^n + f_e\bar{B}^l] \\ \beta_b\alpha_b[f_{b^l1}(\cdot)(-r/q^2)W_h^n + f_{b^l}\bar{E}] & \beta_b\alpha_b[f_{b^l2}(\cdot)(-1/p_b^2)W_h^n + (f_{b^l}-1)\bar{B}^l] \end{pmatrix}$$

The trace of this matrix is obviously negative, while, for the determinant, we obtain the expression:

$$J = \beta_e\alpha_e\beta_b\alpha_b \begin{vmatrix} -f_{e1}(\cdot)r/q^2W_p^n + (f_e-1)\bar{E} & -f_{e2}(\cdot)/p_b^2W_h^n + f_e\bar{B}^l \\ -f_{b^l1}(\cdot)r/q^2W_h^n + f_{b^l}\bar{E} & -f_{b^l2}(\cdot)/p_b^2W_h^n + (f_{b^l}-1)\bar{B}^l \end{vmatrix}$$

We have:

$$-(f_{e1}+f_{b^l1})(\cdot)r/q^2W_h^n + (f_e+f_{b^l}-1)\bar{E} = f_{b1}(\cdot)r/q^2W_h^n - f_b\bar{E} < 0$$

and:

$$-(f_{e2}+f_{b^l2})(\cdot)/p_b^2W_h^n + (f_e+f_{b^l}-1)\bar{B}^l = f_{b2}(\cdot)/p_b^2W_h^n - f_b\bar{B}^l < 0$$

and, thus, get that the negative entries in the diagonal dominate the positive entries in the off-diagonal. This implies that the determinant of J must be positive and, thus, proves the validity of the Routh–Hurwitz stability conditions for such a planar dynamical system. □

Proof of Proposition 2:

The Jacobian of the full 3D system at the steady state is given by:

$$\begin{aligned} J &= \begin{pmatrix} \beta_y(a_y-1) & \beta_y a_q & 0 \\ \beta_e\alpha_e f_{e1}r'/qW_h^n & \beta_e\alpha_e[-f_{e1}r/q^2W_h^n+(f_e-1)\bar{E}] & \beta_e\alpha_e[-f_{e2}/p_b^2W_h^n+f_e\bar{B}^l] \\ \beta_b\alpha_b f_{b^l1}r'/qW_h^n & \beta_b\alpha_b[-f_{b^l1}r/q^2W_h^n+f_{b^l}\bar{E}] & \beta_b\alpha_b[-f_{b^l2}/p_b^2W_h^n+(f_{b^l}-1)\bar{B}^l] \end{pmatrix} \\ &= \begin{pmatrix} - & + & 0 \\ + & - & + \\ - & + & - \end{pmatrix} \end{aligned}$$

The trace of J is obviously negative, and the principal minor of order two:

$$J_1 = \begin{vmatrix} \beta_e\alpha_e[-f_{e1}r/q^2W_h^n+(f_e-1)\bar{E}] & \beta_e\alpha_e[-f_{e2}/p_b^2W_p^n+f_e\bar{B}^l] \\ \beta_b\alpha_b[-f_{b^l1}r/q^2W_h^n+f_{b^l}\bar{E}] & \beta_b\alpha_b[-f_{b^l2}/p_b^2W_h^n+(f_{b^l}-1)\bar{B}^l] \end{vmatrix}$$

is positive according to the above proposition, as is the principal minor, J_2. Additionally, for the remaining principal minor of order two, we get:

$$\begin{aligned} J_3 &= \begin{vmatrix} \beta_y(a_y-1) & \beta_y a_q \\ \beta_e\alpha_e f_{e1}r'/qW_h^n & \beta_e\alpha_e[-f_{e1}r/q^2W_h^n+(f_e-1)\bar{E}] \end{vmatrix} \\ &= \beta_y\beta_e\alpha_e f_{e1}/qW_h^n \begin{vmatrix} a_y-1 & a_q \\ r' & -r/q \end{vmatrix} + \beta_y(a_y-1)(f_e-1)\bar{E} \end{aligned}$$

which is positive if the speed parameter, β_e, is chosen as sufficiently small. Note, however, that the Routh–Hurwitz conditions only demand that the sum of principal minors of order two is to be positive, which provides a much weaker condition than the one just stated.

For the determinant of the Jacobian, J, one gets from the above:

$$\begin{aligned} |J| &= \beta_y\beta_e\alpha_e\beta_b\alpha_b \begin{vmatrix} a_y-1 & a_q & 0 \\ f_{e1}r'/qW_h^n & -f_{e1}r/q^2W_h^n+(f_e-1)\bar{E} & -f_{e2}/p_b^2W_h^n+f_e\bar{B}^l \\ f_{b^l1}r'/qW_h^n & -f_{b^l1}r/q^2W_h^n+f_{b^l}\bar{E} & -f_{b^l2}/p_b^2W_h^n+(f_{b^l}-1)\bar{B}^l \end{vmatrix} \\ &= J_3J_{33} + \beta_y\beta_e\alpha_e\beta_b\alpha_b(f_{e2}/p_b^2W_h^n - f_e\bar{B}^l) \begin{vmatrix} a_y-1 & a_q \\ f_{b^l1}r'/qW_h^n & -f_{b^l1}r/q^2W_h^n+f_{b^l}\bar{E} \end{vmatrix} \end{aligned}$$

In order to get a negative determinant, we, therefore, have to show that the determinant:

$$\begin{vmatrix} a_y-1 & a_q \\ f_{b^l1}r'/qW_h^n & -f_{b^l1}r/q^2W_h^n+f_{b^l}\bar{E} \end{vmatrix}$$

is positive, in addition to the already assumed positivity of the minor, J_3. The last expression here shows that this *for example* holds if the marginal propensity to purchase goods, $a_y \in (0,1)$, is sufficiently close to one.

The condition $(-trJ)(J_1 + J_2 + J_3) - |J| > 0$ can be fulfilled by choosing the adjustment speed of the dynamic multiplier process as sufficiently large, since it enters the product term with power two and the determinant only in a linear form. □