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A Mathematical Model of Global Equity for Allocating Asteroid Resources Based on Analytic Hierarchy Process

Shuyang Qian

Department of Management, Shanghai University, Shanghai, China
Email: junren751113@126.com

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Abstract

As the Earth's resources are utilized, we are increasingly seeking access to space-based resources. One of the most promising solutions to the problem of resource scarcity is asteroid mining. However, it brings with it the problem of resource allocation. Because of the different levels of development of countries, the way of equal distribution is not reasonable. In this paper, we firstly elaborate and define the abstract global equity in a concrete way, linking global equity with comprehensive national strength, and build a new global equity model on this basis. In this way, we can apply the established model to the distribution of space-based resources or other resources. This paper first establishes the corresponding relation between global equity and comprehensive national power, and then determines several important indexes affecting comprehensive national power according to Klein equation. Further, this paper selected four representative large countries and determined the data of each country in each important index by consulting relevant materials. On this basis, the analytic hierarchy process was used to determine the weight of each country in resource allocation. By comparing the weight index obtained by the model in this paper with the actual resource allocation ratio, we can find that they are in good agreement [1]. Therefore, it can be concluded that when allocating space-based resources or other resources, we can use the global equity model established in this paper to calculate the weight index and allocate resources on this basis to ensure the realization of global equity [2].

Keywords

Hierarchical Analysis, Global Equity Resource, Allocation Mod

1. Introduction

The United Nations in 1976 the UN outer space treaty, so far, most of the world's countries have signed the treaty, agreed to the "exploration and use of outer space, including the moon and other celestial bodies, the interests and rights for all countries, regardless of their economic and scientific development, outer space should be the land of mankind" [3]. Research suggests that there are more than 16,000 asteroids with orbits like Earth's near Earth, many of which contain resources that are increasingly scarce on Earth [4]. Therefore, by exploiting the resources of asteroids, humans can relieve the shortage of resources caused by the rapid development of heavy industry on Earth [5]. However, due to the difficulty of exploiting resources in outer space and the possible waste of resources caused by blind exploitation, [6] it is necessary for us to unify opinions within the Earth, formulate corresponding mining strategies and carry out reasonable and fair distribution on this basis, so as to increase global equity and promote the overall development of the human community [7]. In this paper, the analytic hierarchy process is mainly used to measure global equity. By selecting several important indicators and representative countries, the analytic hierarchy process model is used to obtain the proportion of resources allocated by corresponding countries, which can better meet the overall requirements of global equity.

2. Global Equity Model

2.1. Background of Model

Global equity refers to distributing resources according to the contribution of different countries to the world, not evenly distributed in the general sense. In brief, what you deserve matches what you get. We use comprehensive national power [8] to reflect global equity because countries with greater comprehensive national power contribute more to the global economy, in politics, no matter in economics, science or technology. For example, the US and China account for a high proportion of GDP in each country or region to global GDP, at around 23.92% and 18.45%. But India accounts for 3.22% then India's contribution to the world is lower relative to the U.S. and China. On the other hand, when a country achieves outstanding results, other countries will follow it and thus promoting global development. We believe that the "fairness" is "what you deserve matches what you get" so the country that contributes more deserves more resources when it comes to resource allocation.

After the 1970s, a scholar of international issues and strategic studies, R.S. Klein of the United States, conducted a more extensive and in-depth exploration in the quantitative studies of comprehensive national power. In his three works *Evaluation of World Power*, *Evaluation of World Power* 1977 and *World Power Trends and U.S. Foreign Policy in the* 1980*s* published in 1975, 1977 and 1980 respectively, he proposed a mathematical model for measuring and assessing a country's national power based on the kernel of the recent geopolitical doctrine of national power: $P_N = (C + E + W) \times (S + W)$. P_N is the composite national power index; C is the basic entity, consisting of population and land area; E is the economic power, consisting of six categories of indicators, including GDP, energy, critical non-fuel minerals, industrial production capacity, food production, and foreign trade; M is the military power, expressed as the sum of strategic nuclear forces and conventional forces possessed by a country; S is the power of a country conventional forces; S is strategic objectives, which refers to the political goals to be achieved and national interests to be protected in the international environment; W is the will to pursue national strategy, which refers to a country's ability to mobilize its citizens to support the government's defense and foreign policy. As the pioneer of national power measurement, Klein equation has great authority and reference value. So, we choose four important factors in Klein equation as our evaluation index, which are population, land area, GDP, and military power. They account for the national power calculation weights of 10%, 10%, 40%, and 40%, respectively (There is no influence factor brought by science and technology in Klein's equation, then we decided that the science and technology factor has been reflected in the economic power and military power side-by-side because of the wide range of factors involved and influenced by science and technology, and the degree of change of science and technology is so fast due to historical development that it is more complicated and inaccurate to strip the weights separately).

We visualized the data:

China and India have large differences in population size from the other countries and are more dominant in terms of population size; as shown in Figure 1.

Russia, Canada, China, and the United States account for a large portion of the world's land area, and these countries are more dominant in terms of regional size to assess national power; as shown in Figure 2.

The United States, Japan's national GDP is bigger, then the economic strength is higher, the national strength of these countries more dominant; as shown in Figure 3.

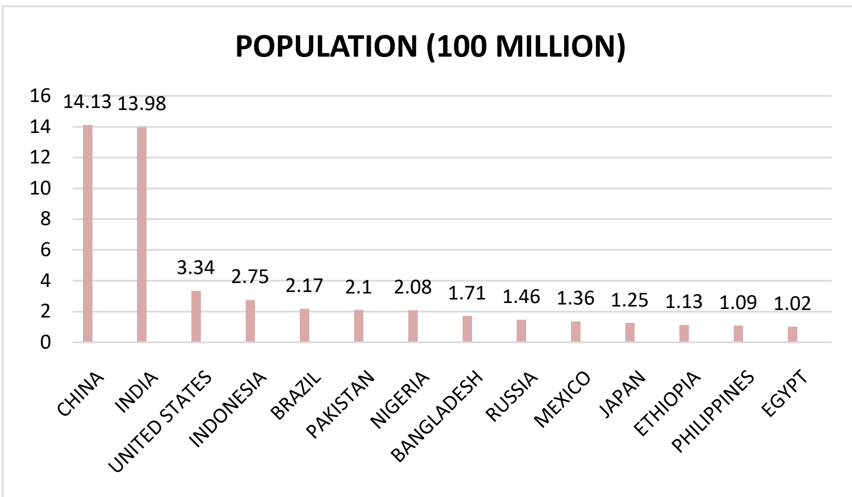

Figure 1. Population size of selected countries.

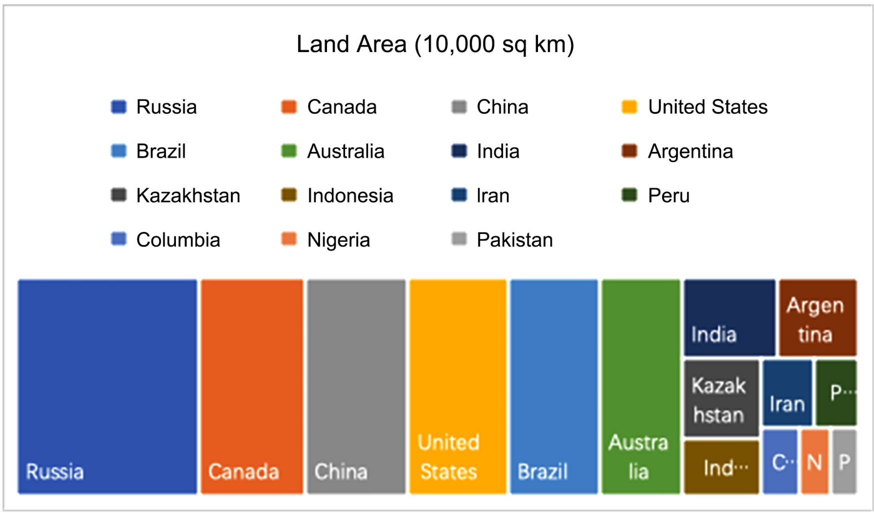

Figure 2. Selected countries' land area.

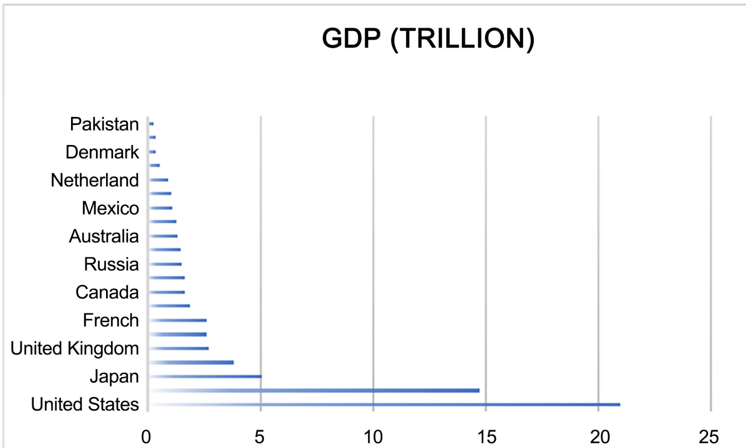

Figure 3. Selected Country GDP.

The lower the ratio, the stronger the military power. If the United States, Russia, and China have more military power, then the judging of national power will be more dominant; as shown in **Figure 4**.

2.2. Build the Hierarchical Model

Analytic hierarchy process (AHP) is a decision analysis method which combines qualitative and quantitative methods to solve multi-objective complex problems. The method uses the decision maker's experience to determine the relative importance between the criteria for the achievability of each measure of the objectives. The proportion of the country power calculation weights has been determined, and a comparative value for these four trade-off indicators that allows for fair feedback in each country needs to be determined.

The decision problem is now decomposed into three levels, the top level is the objective level M, which is to develop a reasonable global equity evaluation system, the middle level is the criterion level, including population C1, land area

C2, GDP C3, and military power C4, and the bottom level is the program level, which is the individual countries with global autonomy, indicated by P1, P2…Pn. To simplify the hierarchical model and control the ratio b greater than 0 and less than 10, we choose the United States, China, India, and Pakistan as representative countries for the calculation of their comprehensive national power [9]; as shown in Figure 5 below.

2.3. Model Solution

2.3.1. Construction of Judgment Matrix M-C

The four elements C1, C2, C3, C4 in the criterion layer C are compared two by two to obtain the pairwise comparison matrix; as shown in Table 1 below.

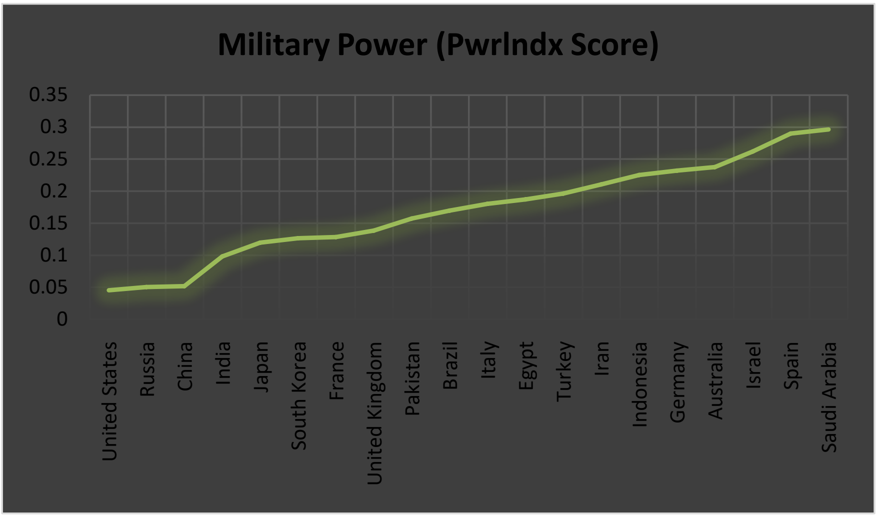

Figure 4. Selected countries' military power.

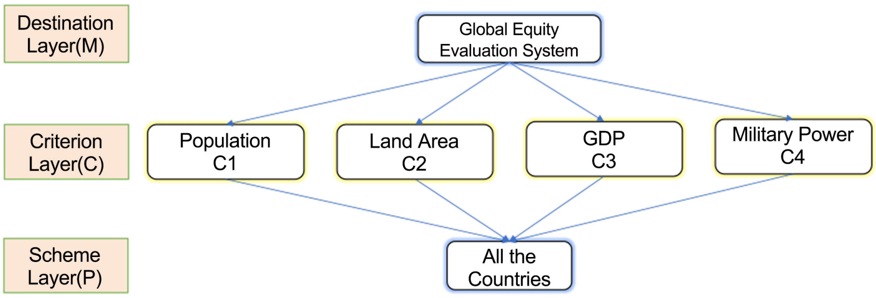

Figure 5. Hierarchical Analysis.

Table 1. Comparison matrix.

M	C1	C2	C3	C4
C1	1.0000	1.0000	0.2500	0.2500
C2	1.0000	1.0000	0.2500	0.2500
C3	4.0000	4.0000	1.0000	1.0000
C4	4.0000	4.0000	1.0000	1.0000

If the positive reciprocal inverse matrix satisfies $a_{ij} \times a_{jk} = a_{ik}$, then we call it a consistent matrix.

Consistency test: 1) Calculate the consistency index CI: $\mathrm{CI} = \frac{\lambda_{\max} - n}{n-1}$.

2) Find the corresponding average random consistency index RI; as shown in Table 2 below.

3) Calculate the consistency ratio CR: $\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}}$. If CR < 0.1, the consistency of the judgment matrix can be considered acceptable, otherwise, the judgment matrix needs to be revised.

Solving for the eigenvalues of M-C and solving for $\lambda_{\max}$ = 4.0000 calculated CR = 0 < 0.1, which passed the consistency test.

2.3.2. Construction of Judgment Matrices C1-P, C2-P, C3-P, C4-P [10]; As Shown in Tables 3-6 below

$\lambda_{\max}$ = 4.0006, CR_1 = 0.0002 < 0.1, $\mathrm{CR}_{2,3,4}$ = 0.0000 < 0.1, by calculation, which passed the consistency test [11].

Table 2. Relationship between n and RI.

n	2	3	4	5	6	7	8	9	10
RI	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46	1.49

Table 3. C1-P judgment matrix.

Population	United States	India	China	Pakistan
United States	1.0000	0.2266	0.2315	1.2247
India	4.4138	1.0000	0.9591	6.7431
China	4.3205	1.0426	1.0000	7.0323
Pakistan	0.6145	0.1483	0.1422	1.0000

Table 4. C2-P judgment matrix.

GDP	United States	India	China	Bakkies
United States	1.0000	7.4350	1.3000	8.4317
India	0.1345	1.0000	0.1749	1.1341
China	0.7691	5.7182	1.0000	6.4851
Pakistan	0.1186	0.8818	0.1542	1.0000

Table 5. C3-P judgment matrix.

Land Area	United States	India	China	Pakistan
United States	1.0000	2.8482	0.9753	9.3245
India	0.3511	1.0000	0.3413	3.2755
China	1.0253	2.9303	1.0000	9.5643
Pakistan	0.1072	0.3053	0.1045	1.0000

Table 6. C4-P judgment matrix.

Military Power	United States	India	China	Pakistan
United States	1.0000	0.4627	0.8865	0.2882
India	2.1611	1.0000	1.9157	0.6227
China	1.1280	0.5220	1.0000	0.3251
Pakistan	3.4702	1.6058	3.0764	1.0000

2.3.3. Calculate Weights

To ensure the robustness of the results, we used the arithmetic mean, geometric mean, and eigenvalue methods to find the weights separately, and then calculated the average value, and then calculated the scores of each scheme based on the obtained weight matrix, which avoids the bias arising from using a single method and the conclusions drawn will be more comprehensive and valid [12]. w_k is the weight share of each influencing factor in each country.

Normalization process: find in the four judgment matrices:

$$w_k = \frac{a_{ij}}{\sum_{i=1}^{4} a_{ij}} \left(k, i, j = 1, 2, 3, 4\right) \tag{1}$$

n-dimensional matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \tag{2}$$

Arithmetic averaging:

$$\omega_i = \frac{1}{n} \sum_{j=1}^{n} \frac{a_{ij}}{\sum_{k=1}^{n} a_{kj}} \left(i = 1, 2, \cdots, n\right) \tag{3}$$

Geometric averaging

$$\omega_i = \frac{\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}}{\sum_{k=1}^{n} \left(\prod_{j=1}^{n} a_{kj}\right)^{\frac{1}{n}}} \left(i = 1, 2, \cdots, n\right) \tag{4}$$

Eigenvalue method: Following the method of consistent matrix weights, we find the maximum eigenvalue of matrix A and the corresponding eigenvector and normalize the found eigenvector to get our weights [13]; as shown in **Tables 7-10** below.

2.3.4. Testing of the Model

Calculate the country's score based on the table above, using the United States as an example: 0.1 × 0.0947 + 0.4 × 0.4945 + 0.1 × 0.40254 − 0.4 × 0.12889 = 0.1960. In this indicator of military power is the opposite, the higher the value, the worse the military power, so we take the subtraction to indicate the impact of military power on the calculation of the overall national power, therefore, the United States 0.1684, India −0.0286, China 0.1597, Pakistan −0.1368. This is consistent with the actual situation, which can prove the validity of the model [14].

Table 7. Arithmetic average method of calculating weights.

		United States	India	China	Pakistan	
Population	0.1	0.0965	0.4182	0.4271	0.0611	1
GDP	0.1	0.4945	0.0665	0.3803	0.1210	1
Land Area	0.4	0.4025	0.1412	0.4131	0.0427	1
Military Power	0.4	0.1289	0.2785	0.1454	0.4472	1

Table 8. Geometric averaging method to calculate weights.

		United States	India	China	Pakistan	
Population	0.1	0.0911	0.4183	0.4294	0.0610	1
GDP	0.1	0.4944	0.0665	0.3803	0.0586	1
Land Area	0.4	0.4025	0.1412	0.4131	0.0431	1
Military Power	0.4	0.1288	0.2785	0.1453	0.4472	1

Table 9. Eigenvalue method for calculating weights.

		United States	India	China	Pakistan	
Population	0.1	0.0966	0.4265	0.4175	0.0594	1
GDP	0.1	0.4945	0.0665	0.3803	0.0587	1
Land Area	0.4	0.4026	0.1414	0.4128	0.0432	1
Military Power	0.4	0.1289	0.2785	0.1454	0.4472	1

Table 10. Indicator weighting table.

		United States	India	China	Pakistan
Population	0.1	0.0947	0.4210	0.4247	0.0605
GDP	0.1	0.4945	0.0665	0.3803	0.0794
Land Area	0.4	0.4025	0.1413	0.4130	0.0430
Military Power	0.4	0.1288	0.2785	0.1453	0.4472

Ebraic regression system is $y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + b$, where y is the explanatory variable, *i.e.*, the dependent variable; x_1-x_4 is the explanatory variable, *i.e.*, the independent variable; a_0 is the regression constant; a_1-a_2 is the regression coefficient; and b is the random error [15].

3. Conclusion

In this paper, we define global equity and develop a global equity model using hierarchical analysis. And we verified the validity of the model. We hope that when asteroid mining becomes a reality, the problem of resource allocation can be properly dealt with, and the fairness between countries and people can be truly achieved.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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