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On Fermat Last Theorem: The New Efficient Expression of a Hypothetical Solution as a Function of Its Fermat Divisors

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Abstract

Denote by (a,b,c) a non-trivial primitive solution of Fermat's equation $(E_F): x^p + y^p = z^p$, $p > 2$ (p prime).We introduce, for the first time, what we call Fermat principal divisors (d,e,f) of the triple (a,b,c) defined as follows. $d = \gcd(a, c-b)$, $e = \gcd(b, c-a)$ and $f = \gcd(c, a+b)$. We show that it is possible to express a,b and c as function of the Fermat principal divisors. Denote by F_p the set of possible non-trivial solutions of the Diophantine equation (E_F). And, let $(a,b,c) \in F_p$, $p > 2$ (p prime). We prove that, in the first case of Fermat's theorem, one has $abc \not\equiv 0\,[p] \Rightarrow \begin{cases} 2a = d^p - e^p + f^p \\ 2b = -d^p + e^p + f^p \\ 2c = d^p + e^p + f^p \end{cases}$.

In the second case of Fermat's theorem, we show that

$$a \equiv 0\,[p] \Rightarrow \begin{cases} 2a = \dfrac{d^p}{p} - e^p + f^p \\ 2b = -\dfrac{d^p}{p} + e^p + f^p \\ 2c = \dfrac{d^p}{p} + e^p + f^p \end{cases}, \quad b \equiv 0\,[p] \Rightarrow \begin{cases} 2a = d^p - \dfrac{e^p}{p} + f^p \\ 2b = -d^p + \dfrac{e^p}{p} + f^p \\ 2c = d^p + \dfrac{e^p}{p} + f^p \end{cases},$$

$$c \equiv 0\,[p] \Rightarrow \begin{cases} 2a = d^p - e^p + \dfrac{f^p}{p} \\ 2b = -d^p + e^p + \dfrac{f^p}{p} \\ 2c = d^p + e^p + \dfrac{f^p}{p} \end{cases}.$$

Furthermore, we have implemented a python program to calculate the Fermat divisors of Pythagoreans triples. The results of this program, confirm the model used. We now have an effective tool to directly process Diophantine equations and that of Fermat.

Keywords

Fermat's Last Theorem, Fermat Divisors, Barlow's Relations, Greatest Common Divisor

1. Introduction

Fermat's Last Theorem (FLT) has fascinated and stimulated many professional and amateur mathematicians. This theorem inspired several authors who have seen in this problem many hidden mysteries waiting to be unveiled [1] [2] [3]. The desire of finding the proof of the Fermat Last Theorem has developed several branches of mathematics as modern number theory [4]-[9]. However, after the proof of Wiles in 1995, this desire dropped considerably [8]. Several paths have been taken to solve FLT and some of its sub-problems were abandoned. Consequently, there are sub-problems that have not been fully resolved by direct means. Either for the reason mentioned above or for the lack of efficient tools. We can cite some of these problems:

In the first case, we have the following results with different hypotheses.

- If $z = y+1$, FLT holds, then the first case of Abel conjecture is also proved. [1] (p.196), Tab.A;
- For $y = x+1$, FLT has been proved by Dittmann [1] (p.201);
- In 1823, Sophie Germain has demonstrated FLT with $2p+1$ a prime (pp. 109-112) [1] [10];
- In 1977, Terjanian proved FLT for $n = 2p$ (p. 209) [1] [11].

However, the second case of those results has not been directly proved. Note that some intermediate problems related to the FLT have not been stated or treated. For example, in the first case, we have:

- If FLT is false, $y-x, x+z, y+z$ are not divisible by *p*.
- If FLT is false for the exponent *p*, then $2^p \equiv 1\left[p^3\right]$.
- A direct proof that FLT is true for the non-prime odd exponent.

In this paper, we give efficient tools to solve these problems mentioned above and a direct proof of Fermat Last Theorem. So, we prove the following results.

Theorem 1. Let $p > 2$ (p prime) and $(a,b,c) \in F_p$ be a primitive triple with (d,e,f) its principal divisors. We have:

$$abc \not\equiv 0\left[p\right] \Rightarrow \begin{cases} 2a = d^p - e^p + f^p \\ 2b = -d^p + e^p + f^p \\ 2c = d^p + e^p + f^p \end{cases} \text{ with } d = \gcd(a, c-b).$$

Theorem 2. Let $p > 2$ (p prime) and $(a,b,c) \in F_p$ be a primitive triple with (d,e,f) its principal divisors. We have:

$$a \equiv 0[p] \Rightarrow \begin{cases} 2a = \dfrac{d^p}{p} - e^p + f^p \\ 2b = -\dfrac{d^p}{p} + e^p + f^p \\ 2c = \dfrac{d^p}{p} + e^p + f^p \end{cases}, \quad b \equiv 0[p] \Rightarrow \begin{cases} 2a = d^p - \dfrac{e^p}{p} + f^p \\ 2b = -d^p + \dfrac{e^p}{p} + f^p \\ 2c = d^p + \dfrac{e^p}{p} + f^p \end{cases},$$

$$c \equiv 0[p] \Rightarrow \begin{cases} 2a = d^p - e^p + \dfrac{f^p}{p} \\ 2b = -d^p + e^p + \dfrac{f^p}{p} \\ 2c = d^p + e^p + \dfrac{f^p}{p} \end{cases}$$

We prove these main results based on Fermat divisors in the following plan: First, preliminary and second, the proofs of the theorems.

2. Preliminary

2.1. The Fermat Divisors

In this sub-section, we introduce for the first time the Fermat divisors associated with a hypothetical solution of the equation E_H. Then, we effectively compute the Fermat divisors in the case of the Pythagorean equation where we call them Pythagorean divisors. Fermat's divisors are efficient tools in the implementation of a direct algebraic proof of FLT. Here, we are laying the foundations of these tools.

Definition 2.1. Let $p > 2$ (p prime), $(a,b,c) \in F_p$ a primitive triple, $d = \gcd(a, c-b)$, $e = \gcd(b, c-a)$, $f = \gcd(c, a+b)$, $a = d\alpha$, $b = e\beta$ and $c = f\gamma$.

The triples (d,e,f) and (α,β,γ) are respectively defined as primary divisors and secondary divisors of Fermat associated to the triple (a,b,c).

Example 2.1. Let F_2 be the set of non-trivial primitive Pythagorean triples of positive integers. Some of these divisors are illustrated very well with Pythagorean triples. In this case, we speak about Pythagorean divisors. The following table gives the values of these divisors for some Pythagorean triples. The data for Table 1, is obtained by running the python program in Figure 1.

Lemma 2.1. Let $(a,b,c) \in F_2$ a primitive triple.

$$f = \gcd(c, a+b) \Rightarrow f^2 = \gcd\left(c^2, (a+b)^2\right)$$

Proof.

$$\begin{aligned} f = \gcd(c, a+b) &\Rightarrow c = fq \text{ and } a+b = fr \text{ and } \gcd(q,r) = 1 \\ &\Rightarrow c^2 = f^2q^2 \text{ et } (a+b)^2 = f^2r^2 \text{ et } \gcd(q,r) = 1 \\ &\Rightarrow \gcd\left(c^2, (a+b)^2\right) = \gcd\left(f^2q^2, f^2r^2\right) \\ &\Rightarrow \gcd\left(c^2, (a+b)^2\right) = f^2 \gcd\left(q^2, r^2\right) \end{aligned}$$

$$\Rightarrow \gcd\left(c^2,(a+b)^2\right) = f^2 \text{ because } \gcd\left(q^2,r^2\right) = 1$$

Proposition 2.1. Let (a,b,c) a primitive triple.

$$(a,b,c) \in F_2 \Rightarrow f = 1 \text{ and } \gamma = c.$$

Proof:

Suppose that $f > 1$, then we have:

$$f = \gcd(c,a+b) \Rightarrow f^2 = \gcd\left(c^2,(a+b)^2\right)$$

$$\Rightarrow f^2 = \gcd\left(a^2+b^2,(a+b)^2\right)$$

$$\Rightarrow f^2 = \gcd\left(a^2+b^2,a^2+b^2+2ab\right)$$

$$\Rightarrow 2ab \equiv 0\left[f\right]$$

$$\Rightarrow ab \equiv 0\left[f\right] \text{ because } f \text{ is odd}$$

$$\Rightarrow \square \quad \text{because} \quad \gcd(ab,c) = 1$$

Thus, we indeed have $f = 1$ so $c = \gamma f = \gamma$ as announced.

Remark 2.1. For any primitive triples $(a,b,c) \in F_2$, the divisors $(d,e,1)$ and $(\alpha,\beta,c=\gamma)$ are defined as Pythagorean divisors.

```
Import math
def CalcPythaDiv(M,N):
    for a in range(M,N):
        for b in range(a,math.floor((a/2+1)*a)):
            for c in range(b+1, a+b):
                if(math.gcd(a,b)==1and math.gcd(a,c)==1 and a*a+b*b==c*c):
                    d=math.gcd(a,c-b);e=math.gcd(b,c-a); f=math.gcd(c,a+b);
                    alpha=a//d; beta=b//e; gamma=c//f
                    print(a,b,c,d,e,f,alpha,beta,gamma)
```

Figure 1. Python programs to compute pythagorean triplets and theirdivisors.

Table 1. Examples of Fermat's divisors based on Pythagorean triples. Extract from the results of the command CalcPythaDiv (3, 40).

$(a,b,c) \in F_2$			d	e	f	α	β	γ
3	4	5	1	2	1	3	2	5
5	12	13	1	4	1	5	3	13
15	8	17	3	2	1	5	4	17
20	21	29	4	3	1	5	7	29
20	99	101	2	9	1	10	11	101
28	45	53	4	5	1	7	9	53
35	12	37	5	2	1	7	6	37

2.2. Terjanian Polynomial and Some Properties

Definition 2.2. Let $n > 2$ an integer. We define the Terjanian polynomial of degree *n*, as follow

$$T_n(x,y) = \frac{y^n - x^n}{y - x} \text{ with } y \neq x.$$

Remark 2.2. Note that

$$T_n(x,y) = \sum_{k=0}^{n-1} C_n^k (y-x)^{n-k-1} x^k$$

Lemme 2.2. Let $p > 2$ a prime.

$$(a,b,c) \in F_p \Rightarrow T_p(a,b) = (b-a)^{p-1} + \sum_{k=1}^{p-1} C_p^k (b-a)^{p-1-k} a^k.$$

Proof.

$$(a,b,c) \in F_p \Rightarrow b^p - a^p = (b-a+a)^p - a^p$$

$$\Rightarrow (b-a)T_p(a,b) = (b-a)^p + \sum_{k=1}^{p-1} C_p^k (b-a)^{p-k} a^k$$

$$\Rightarrow T_p(a,b) = (b-a)^{p-1} + \sum_{k=1}^{p-1} C_p^k (b-a)^{p-1-k} a^k$$

$$\Rightarrow T_p(a,b) = (b-a)^{p-1} + \sum_{k=1}^{p-2} C_p^k (b-a)^{p-1-k} a^k + pa^{p-1}$$

Lemme 2.3. Let $p > 2$ a prime and $(a,b,c) \in F_p$.

$$abc \not\equiv 0[3] \Rightarrow b - a \equiv 0[3]$$

Proof. Let $p > 2$ and $(a,b,c) \in F_p$. We have:

$$abc \not\equiv 0[3] \Rightarrow a^p + b^p = c^p$$

$$\Rightarrow a + b = c[3] \text{ because } p \text{ is odd}$$

$$\Rightarrow (b-a)(a+b) = (b-a)c[3]$$

$$\Rightarrow b^2 - a^2 \equiv (b-a)c[3]$$

$$\Rightarrow 0 \equiv (b-a)c[3]$$

$$\Rightarrow b - a \equiv 0[3] \; \textit{because} \; c \not\equiv 0[3]$$

Proposition 2.2. Let $(a,b,c) \in F_p$,

$$abc \not\equiv 0[3] \Rightarrow T_p(a,b) \equiv p[3]$$

Proof. Let $(a,b,c) \in F_p$ and $abc \not\equiv 0[3]$

$$b - a \equiv 0[3] \Rightarrow T_p(a,b) \equiv pa^{p-1}[3] \text{ [lemmas 2.2 \& 2.3]}$$

$$\Rightarrow T_p(a,b) \equiv p[3]$$

$$\Rightarrow \square$$

Proposition 2.3. Let $p > 2$ a prime, $(a,b,c) \in F_p$ and $abc \not\equiv 0[p]$

$$b - a \equiv 0[p] \Rightarrow T_p(a,b) \equiv p\left[p^2\right]$$

Proof. Let $p > 2$ a prime, $(a,b,c) \in F_p$ and $abc \not\equiv 0[p]$.

$$b - a \equiv 0[p] \Rightarrow T_p(a,b) \equiv pa^{p-1}\left[p^2\right] \text{ [lemma 2.2]}$$

$$\Rightarrow T_p(a,b) \equiv p\left[p^2\right] \text{ because } a^{p-1} \equiv 1\left[p^2\right] \text{ ([1], p. 167)}$$
$$\Rightarrow \square$$

2.3. First Properties of Fermat Divisors

In this section, we prove the following relations which are like Barlow's relations [4, p.100] for a hypothetical primitive solution (a,b,c) of Fermat's equation.

Lemma 2.4. Let $(a,b,c) \in F_p$ a primitive triple. We have:

$$abc \not\equiv 0[p] \Rightarrow def \not\equiv 0[p]$$

Proof. Suppose that $def \equiv 0[p]$ and $d \equiv 0[p]$. Then $a \equiv 0[p]$ so $abc \equiv 0[p]$. That contradicts the assumption $abc \not\equiv 0[p]$. We have the same results with the cases $e \equiv 0[p]$ and $f \equiv 0[p]$.

Lemma 2.5. Let $p > 2$ a prime and $(a,b,c) \in F_p$ a primitive triple with $(d,e,f,\alpha,\beta,\gamma)$ its Fermat divisors. We have:

$$abc \not\equiv 0[p] \Rightarrow \begin{cases} c-b = d^p, T_p(b,c) = \alpha^p, a = d\alpha \\ c-a = e^p, T_p(a,c) = \beta^p, b = e\beta \\ a+b = f^p, T_p(-a,b) = \gamma^p, c = f\gamma \end{cases}.$$

Proof. Firstly,

$(a,b,c) \in F_p \Rightarrow a^p = c^p - b^p \Rightarrow a^p = (c-b)T_p(b,c)$ and $\gcd(c-b, T_p(b,c)) = 1$

$$\Rightarrow d^p\alpha^p = (c-b)T_p(b,c) \text{ and } \gcd(c-b, T_p(b,c)) = 1$$

Where, because $c-b \equiv 0[d]$ and $\gcd(c-b,\alpha) = 1$

$$d^p\alpha^p = (c-b)T_p(b,c) \textit{ and } \gcd(c-b, T_p(b,c)) = 1 \Rightarrow c-b \equiv 0\left[d^p\right]$$

and,

$$d^p\alpha^p = (c-b)T_p(b,c) \textit{ and } \gcd(c-b, T_p(b,c)) = 1 \Rightarrow d^p \equiv 0[c-b]$$

so

$$d^p\alpha^p = (c-b)T_p(b,c) \textit{ and } \gcd(c-b, T_p(b,c)) = 1 \Rightarrow c-b = d^p.$$

Secondly,

$$\begin{aligned} (a,b,c) \in F_p \textit{ and } c-b > 1 &\Rightarrow c-b = d^p \textit{ and } a^p = c^p - b^p \\ &\Rightarrow c-b = d^p \textit{ and } d^p\alpha^p = (c-b)T_p(b,c) \\ &\Rightarrow d^p\alpha^p = d^pT_p(b,c) \text{ because } c-b = d^p \\ &\Rightarrow \alpha^p = T_p(b,c) \end{aligned}$$

In Fermat's equation, *a* and *b* can play symmetrical roles. The case of $c-a$ is proved in the same way as that of $c-b$. The case of $a+b$ is not completely different. We factorize Fermat's equation as follows

$$(a+b)T_p(-a,b) = c^p,$$

and we proceed in the same way as before.

Remark 2.3. We have

$$\begin{cases} c-a > c-b \\ c-b \geq 1 \end{cases} \Rightarrow c-a > 1 \Rightarrow e > 1.$$

In the second case of this problem, we have the following results.

Lemma 2.6. Let $p>2$ be a prime and $(a,b,c)\in F_p$ a primitive triple with $(d,e,f,\alpha,\beta,\gamma)$ its Fermat divisors. We have:

$$a\equiv 0[p]\Rightarrow\begin{cases}c-b=\dfrac{d^p}{p},\,T_p(b,c)=p\alpha^p,\,a=d\alpha\\ c-a=e^p,\,T_p(a,c)=\beta^p,\,b=e\beta\\ a+b=f^p,\,T_p(a,-b)=\gamma^p,\,c=f\gamma\end{cases}$$

Proof. Let $(a,b,c)\in F_p$

$$\begin{aligned}a\equiv 0[p]&\Rightarrow a^p=c^p-b^p\\ &\Rightarrow d^p\alpha^p=(c-b)T_p(b,c)\ and\ gcd(c-b,T_p(b,c))=p\\ &\Rightarrow p^{vp}d_1^p\alpha^p=p^2\frac{c-b}{p}\times\frac{T_p(b,c)}{p}\\ &\Rightarrow p^{vp-2}d_1^p\alpha^p=\frac{c-b}{p}\times\frac{T_p(b,c)}{p}\\ &\Rightarrow d_1^p\alpha^p=\frac{c-b}{p^{vp-1}}\times\frac{T_p(b,c)}{p}.\end{aligned}$$

As $\gcd\left(\dfrac{c-b}{p^{p-1}},\alpha\right)=1$ and $\gcd\left(\dfrac{T_p(b,c)}{p},d_1\right)=1$, we have firstly

$$d_1^p\alpha^p=\frac{c-b}{p^{p-1}}\times\frac{T_p(b,c)}{p}\Rightarrow\frac{T_p(b,c)}{p}\equiv 0\left[\alpha^p\right]\text{ and }\frac{c-b}{p^{p-1}}\equiv 0\left[d_1^p\right]$$

and secondly

$$d_1^p\alpha^p=\frac{c-b}{p^{p-1}}\times\frac{T_p(b,c)}{p}\Rightarrow\alpha^p\equiv 0\left[\frac{T_p(b,c)}{p}\right]\text{ and }d_1^p\equiv 0\left[\frac{c-b}{p^{p-1}}\right]$$

we deduce that:

$$\begin{aligned}a\equiv 0[p]&\Rightarrow\alpha^p=\frac{T_p(b,c)}{p}\text{ and }d_1^n=\frac{c-b}{p^{p-1}}\\ &\Rightarrow T_p(b,c)=p\alpha^p\text{ and }c-b=p^{vp-1}d_1^p=\frac{p^{vp}d_1^p}{p}\\ &\Rightarrow T_p(b,c)=p\alpha^p\text{ and }c-b=p^{vp-1}d_1^p=\frac{d^p}{p}\end{aligned}$$

In Fermat's equation, a and b can play symmetrical roles, we get the same result with $c-a$ and $a+b$.

Remark 2.4.

An approach like the above allows us to obtain the following results.

$$b\equiv 0[p]\Rightarrow\begin{cases}c-b=d^p,\,T_p(b,c)=\alpha^p,\,a=d\alpha\\ c-a=\dfrac{e^p}{p},\,T_p(a,c)=p\beta^p,\,b=e\beta\\ a+b=f^p,\,T_p(-a,b)=\gamma^p,\,c=f\gamma\end{cases}$$

and,

$$c \equiv 0[p] \Rightarrow \begin{cases} c-b = d^p, T_p(b,c) = \alpha^p, a = d\alpha \\ c-a = e^p, T_p(a,c) = \beta^p, b = e\beta \\ a+b = \dfrac{f^p}{p}, T_p(-a,b) = p\gamma^p, c = f\gamma \end{cases}$$

3. Proofs of the Theorems

3.1. Proof of Theorem 1

In this subsection, we demonstrate theorem 1 mentioned in the introduction.

Proof.

Let $p > 2$ (p prime) and $(a,b,c) \in F_p$ a primitive triple with $(d,e,f,\alpha,\beta,\gamma)$ its Fermat divisors.

$$abc \not\equiv 0[p] \Rightarrow \begin{cases} 2a = c-b-(c-a)+a+b \\ 2b = -(c-b)+(c-a)+a+b \\ 2c = (c-b)+(c-a)+(a+b) \end{cases}$$

$$\Rightarrow \begin{cases} 2a = d^p - e^p + f^p \\ 2b = -d^p + e^p + f^p \\ 2c = d^p + e^p + f^p \end{cases} \quad \text{[lemmes 2.5]}$$

3.2. Proof of Theorem 2

Here, we prove theorem 2 mentioned in the introduction.

Proof

Let $p > 2$ be a prime and $(a,b,c) \in F_p$ a non-trivial primitive triple.

$$b \equiv 0[p] \Rightarrow \begin{cases} 2a = (c-b)-(c-a)+a+b \\ 2b = -(c-b)+(c-a)+a+b \\ 2c = (c-b)+(c-a)+(a+b) \end{cases}$$

$$\Rightarrow \begin{cases} 2a = d^p - \dfrac{e^p}{p} + f^p \\ 2b = -d^p + \dfrac{e^p}{p} + f^p \\ 2c = d^p + \dfrac{e^p}{p} + f^p \end{cases} \quad \text{[Remark 2.4]}$$

Remark 3.1.

Similarly, we show that

$$a \equiv 0[p] \Rightarrow \begin{cases} 2a = \dfrac{d^p}{p} - e^p + f^p \\ 2b = -\dfrac{d^p}{p} + e^p + \dfrac{e^p}{p} + f^p \\ 2c = \dfrac{d^p}{p} + e^p + f^p \end{cases} \quad \text{and} \quad c \equiv 0[p] \Rightarrow \begin{cases} 2a = d^p - e^p + \dfrac{f^p}{p} \\ 2b = -d^p + e^p + \dfrac{f^p}{p} \\ 2c = d^p + e^p + \dfrac{f^p}{p} \end{cases}$$

4. Conclusion

We introduced, for the first time, the Fermat divisors, in the study of the Fermat Last Theorem (FLT). Then, we expressed the eventual solutions in terms of their principal divisors. We have improved and implemented, respectively, Barlow relations and efficient tools for direct evidence of FLT. In the future, we plan to:

- Express Pythagorean triples in terms of its Pythagorean divisors.
- Express hypothetical solution of $x^{2p} + y^{2p} = z^{2p}$ in term of its Fermat divisors.
- We will use these expressions to solve Diophantine equations in general and in particular to prove FLT and its sub-problems as mentioned in the introduction.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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