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Robustness of T-test Based on Skewness and Kurtosis

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors wrote the first draft and the computer code. STG analyzed the results. KMO managed the literature searches. Both authors read and approved the final manuscript.

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Abstract

Coverage probabilities of the two-sided one-sample t-test are simulated for some symmetric and right-skewed distributions. The symmetric distributions analyzed are Normal, Uniform, Laplace, and student-t with 5, 7, and 10 degrees of freedom. The right-skewed distributions analyzed are Exponential and Chi-square with 1, 2, and 3 degrees of freedom. Left-skewed distributions were not analyzed without loss of generality. The coverage probabilities for the symmetric distributions tend to achieve or just barely exceed the nominal values. The coverage probabilities for the skewed distributions tend to be too low, indicating high Type I error rates. Percentiles for the skewness and kurtosis statistics are simulated using Normal data. For sample sizes of 5, 10, 15 and 20 the skewness statistic does an excellent job of detecting non-Normal data, except for Uniform data. The kurtosis statistic also does an excellent job of detecting non-Normal data, including Uniform data. Examined herein are Type I error rates, but not power calculations. We find that sample skewness is unhelpful when determining whether or not the t-test should be used, but low sample kurtosis is reason to avoid using the t-test.

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1 Introduction

A fundamental step for researchers when performing statistical analysis is to examine the shape and spread of their data. The most commonly used tests in statistics require assumptions for the data to be met in order to obtain significant results from the test. For example, when using the Student's t-test with small sample sizes, data typically are required to be approximately Normally distributed [1]. However, the Central Limit Theorem gives more flexibility by allowing researchers to relax the Normality assumption if the sample size is large enough [2]. Researchers use a combination of statistical tests, visual assessments, and knowledge of descriptive statistics to decide whether or not Normality should be assumed [3].

However, many situations occur where researchers are unable to access a sample size large enough to assume approximate Normality of the sample mean. Often, this is the case for psychology research, where data might not be abundant. The Welch's t-test can be used as a competitor to the Student's t-test, where the former is shown to be more accurate when working with some non-Normal distributions [4]. The sign test is also recommended to use when Normality assumptions are violated. However, researchers performed an analysis that showed the t-test is not necessarily less useful when working with non-Normal data [5]. Considering the difficulty in verifying the Normality assumption for small sample sizes, we are interested in determining the robustness of the t-test when the Normal assumptions are not reasonably met. In order to determine a metric for measuring non-Normality, we examine descriptive statistics that give us an insight as to how much a sample deviates from Normality. By considering the skewness and kurtosis of data, researchers and statisticians are able to measure the central tendency and shape of their data [6]. Skewness and kurtosis provide much needed insight for studies, such as their effect on stock market volatility [7] or analyzing test score distributions [8]. However, skewness and kurtosis are sometimes overlooked in elementary statistics courses due to the popularity of more intuitive statistics such as mean and standard deviation [9].

Multivariate skewness is considered in financial time series, and possible future research in multivariate skewness is suggested [10]. Skewness is discussed regarding the one-sample one-tailed Student's t-statistic [11]. Sampling properties of data projections with maximal skewness are investigated, and the sampling behavior of this skewness measure currently is mostly unknown, so future research on this topic is suggested [12]. Theoretical results addressing the role of skewness and kurtosis are discussed in the performance of the one-tailed t-test [13].

Various normality tests are discussed and compared in terms of performance when using non-normal data [14]. The objective herein is to focus on simple measures, symmetry and kurtosis, which elementary statistics students can understand based on graphs. We also focus on a set of distributions, which vary according to skewness and kurtosis, but by no means do we claim that our set of distributions is exhaustive. We fully acknowledge that other distributions and test statistics unrelated to symmetric and kurtosis might provide different results, so we are not attempting to generalize our results to all situations, and those other distributions and test statistics produce grounds for future research. Our research uses 90% confidence intervals, so the benchmark coverage probability is also 90%. However, when coverage deviates much less than 90% (such as 85%), then the test produces too high of Type I error rate; yet, when coverage deviates much greater than 90% (such as 95%), then the test loses power.

Sample sizes of 5, 10, 15, 20, 25 are used to represent the violation of the large sample size requirement. De Winter reported that a high Type I error rate occurs when working with unequal variances and unequal sample sizes for two-sample tests. However, Type II error rates are smaller with small sample sizes if the effect size is large [15].

By using the *R*-package `moments`, we were able to calculate respective skewness and kurtosis values for the simulations. The package `moments` has many different functions which allow users to perform tests, but herein only skewness and kurtosis tests are examined [16]. By generating simulations, we were able to use skewness and kurtosis test statistics to find coverage probabilities in order to test the robustness of the t-test under different distributions and sample sizes.

2 Simulation Approach

A condition when using the one-sample *t*-test or constructing one-sample *t*-confidence intervals on an unknown population mean is to have a large sample size or to have the data sampled from an approximately Normal distribution or a reasonable compromise between having a large sample size and approximately Normal data. Data from Normal, Uniform, Laplace, T_5, T_7, T_{10}, χ^2_1, χ^2_2, χ^2_3, and Exponential distributions are simulated for the small sample sizes of 5, 10, 15, and 20. For each distribution, the one-sample two-sided *t*-test is performed 30 million times with null mean equal to the true population mean, so the parameters (such as the population mean of the Exponential distribution, or the population mean and standard deviation of the Normal distribution) of these distributions are irrelevant. The Laplace distribution is also called the Double Exponential distribution and is symmetric with density

$$\exp\{|x-\mu|\ (\sqrt{2}\ )/\sigma\}\ /\ (\sigma\ \sqrt{2}\ ),\ \forall x \in \Re,$$

where μ and σ are the population mean and standard deviation, respectively.

The symmetric distributions examined herein are the Normal, Uniform, Laplace, and *t*-distributions. The χ^2 and Exponential distributions are right-skewed. The results for right-skewed distributions are applicable to left-skewed distributions using a simple symmetry argument, noting negating values from a right-skewed distribution produces a left-skewed distribution. Thus, left-skewed distributions are not examined herein, without loss of generality.

Pearson's population skewness is defined to be $\kappa_3\ /\ \sigma^3$, where κ_3 is the third population central moment and σ is the population standard deviation. For the Normal, Uniform, Laplace, and *t*-distribution with more than three degrees of freedom, the population skewness is zero. The population skewness for the Exponential distribution is 2; for χ^2_1 is $\sqrt{8} \approx 2.83$; for χ^2_2 is 2; and for χ^2_3 is $\sqrt{8/3} \approx 1.64$;

Pearson's population kurtosis is defined to be $\kappa_4\ /\ \sigma^4$, where κ_4 is the fourth central population moment and σ is the population standard deviation. The population kurtosis for a Normal distribution is 3; for Uniform is 1.8; for T_5 is 9; for T_7 is 5; for T_{10} is 4; for χ^2_1 is 15; for χ^2_2 is 9; for χ^2_3 is 7; for Exponential is 9; and for Laplace is 6.

Although multiple definitions of sample skewness exist in the literature, the common definitions differ by merely a constant multiplicative function of the sample size. Therefore, the particular definition of sample skewness is irrelevant for our purposes. Nevertheless, we used the `skewness` function from the *R*-package `moments`, which defines sample skewness to be $m_3\ /\ m_2^{3/2}$, where m_2 and m_3 are the second and third sample central moments, respectively. Similarly, the particular definition of sample kurtosis is also irrelevant for our purposes. We used the `kurtosis` function from the *R*-package `moments`, which defines sample kurtosis to be $m_4\ /\ m_2^2$, where m_2 and m_4 are the second and fourth sample central moments, respectively.

The sample skewness is unbiased only when the population skewness is zero for the distributions we are analyzing herein. For the other distributions with finite population skewness, the bias of the sample skewness converges to zero as the sample size goes to infinity. For example, the bias of the sample skewness based on the χ_1^2-distribution is approximately -1.54 for sample size $n = 10$ but is approximately -0.87 for $n = 30$, using R-code:

```
> c( mean( replicate( 1e7, skewness( rchisq( 10, 1 ) ) ) ) ,
     mean( replicate( 1e7, skewness( rchisq( 30, 1 ) ) ) ) ) - sqrt( 8 )
```

For the distributions with finite population kurtosis, the bias of the sample kurtosis is nonzero but converges to zero as the sample size goes to infinity. For example, the bias of the sample kurtosis based on the χ_1^2-distribution is approximately -11.24 for $n = 10$ but is approximately -8.00 for $n = 30$, using R-code:

```
> c( mean( replicate( 1e7, kurtosis( rchisq( 10, 1 ) ) ) ) ,
     mean( replicate( 1e7, kurtosis( rchisq( 30, 1 ) ) ) ) ) - 15
```

As another example, the bias of the sample kurtosis based on the Normal distribution is approximately -0.55 for $n = 10$ but is approximately -0.19 for $n = 30$, using R-code:

```
> c( mean( replicate( 1e7, kurtosis( rnorm( 10 ) ) ) ) ,
     mean( replicate( 1e7, kurtosis( rnorm( 30 ) ) ) ) ) - 3
```

Hence, both the sample skewness and sample kurtosis can be heavily biased for small sample sizes.

The 90th percentile of the absolute value of the sample skewness for a Normal distribution was estimated using 30 million simulations. These simulated 90th percentiles of absolute value of skewness are 1.0492502, 0.9537500, 0.8515499, and 0.7718505 for sample sizes of 5, 10, 15, and 20, respectively. The 5th and 95th percentiles of the sample kurtosis for a Normal distribution were estimated, also using 30 million simulations. These simulated 5th percentiles of kurtosis are 1.278078, 1.563894, 1.721885, and 1.830854; and these simulated 95th percentiles of kurtosis are 2.876782, 3.940937, 4.118671, and 4.149351, for sample sizes of 5, 10, 15, and 20, respectively. Therefore, all of the coverage probabilities simulated herein should be compared to the nominal value of 90%. This nominal value 90% was selected, rather than 95%, to keep the simulation error low, noting the standard error of $\sqrt{(p(1-p)/m}$, where p is the coverage probability, and where m is the number of simulations either overall, or for low or high absolute skewness, or for low, medium, or high kurtosis.

Based on Table 1, far-right column, the overall coverage probabilities based on the t-test are close to the nominal value of 90% for all of the sample sizes of 5, 10, 15, and 20, and for all of the symmetric distributions (i.e., Normal, Uniform, T_5, T_7, T_{10}, and Laplace). However, the overall coverage probabilities tend to be too low for the skewed distributions (i.e., χ_1^2, χ_2^2, χ_3^2, and Exponential), especially for the smaller sample sizes. Therefore, the t-test is not recommended for these skewed distributions with small sample sizes.

The computing time for finding these percentiles was 1.7 hours. The computing time for finding coverage probabilities (discussed in the sections below) of the t-test for these various distributions (based on non-high skewness, high skewness, low kurtosis, medium kurtosis, and high kurtosis) was 12 days.

3 Results Based on Skewness

As expected, 10% of the samples from the Normal distribution produce data which meet the high-skewness criteria of 1.0492502, 0.9537500, 0.8515499, and 0.7718505 for sample sizes of 5, 10, 15, and 20, respectively, as noted in the *Proportion high skewness* column in Table 1. In Table 1, the column *Coverage prob high skewness* shows the coverage probabilities of the simulated datasets meeting the high-skewness criteria. The column *Coverage prob low skewness* shows the coverage probabilities of the simulated datasets NOT meeting the high-skewness criteria. The Normal distribution shows coverage probabilities of about 90%, regardless of whether or not the dataset meets the high-skewness criteria. Furthermore, for the T_5, T_7, and T_{10} distributions, the coverage probabilities also are around 90%, regardless of whether or not the dataset meets the high-skewness criteria.

An interesting situation occurs with the Uniform distribution. The coverage probabilities tend to be way too low for the large sample sizes (coverage probability is 0.2446 for $n = 20$) when the high-skewness criteria are met, but the likelihood of meeting the high-skewness criteria is quite low (for $n = 20$, proportion of high skewness is 0.0220, which is well under 0.1).

For the skewed distributions (i.e., χ^2_1, χ^2_2, χ^2_3, and Exponential), the coverage probabilities are surprisingly higher for the datasets meeting the high-skewness criteria than for the datasets NOT meeting the high-skewness criteria. In fact, most of the coverage probabilities for the χ^2_2, χ^2_3 and Exponential distributions are around 90% when the high-skewness criteria is met.

This creates a dilemma. The t-test is anti-conservative for skewed distributions with small sample sizes, as indicated by the coverage overall coverage probabilities less than 90%, in the far-right column of Table 1. However, if sample skewness is used to judge whether or not data appear to be from a skewed distribution, then ironically the coverage probabilities are higher (i.e., closer to 90%) when the datasets meet the high-skewness criteria than when the datasets do NOT meet the high-skewness criteria. Therefore, although approximate Normality typically is a necessary condition for the usual t-test to hold with small sample sizes, disqualifying the t-test due to skewed data is unhelpful, as noted by Table 1 regarding the χ^2_2, χ^2_3, and Exponential distributions.

4 Results Based on Kurtosis

As expected, 5% of the samples from the Normal distribution produce data which meet the low-kurtosis criteria of 1.278078, 1.563894, 1.721885, and 1.830854 for sample sizes of 5, 10, 15, and 20, respectively, as noted in the *Proportion low kurtosis* column in Table 2. Furthermore, as expected, 5% of the samples from the Normal distribution produce data which meet the high-kurtosis criteria of 2.876782, 3.940937, 4.118671, and 4.149351 for sample sizes of 5, 10, 15, and 20, respectively, as noted in the *Proportion high kurtosis* column in Table 2. That leaves 90% of the samples from the Normal distribution producing data which fail to meet either the low-kurtosis criteria or the high-kurtosis criteria, as noted in the *Proportion middle kurtosis* column in Table 2. The Normal distribution shows coverage probabilities of about 90%, regardless of whether or not the dataset meets the low-kurtosis or high-kurtosis criteria.

For the Uniform distribution, the coverage probabilities tend to be slightly low (0.8447 for $n = 20$) when the dataset produces medium kurtosis. When the data from a Uniform distribution produce low kurtosis, which occurs rather frequently (0.4595 for $n = 20$), the t-test is too conservative, producing coverage probabilities between 92% and 97%. Datasets from the Uniform distribution producing high kurtosis are quite rare (as low as 0.0006 for $n = 20$), but such datasets produce very low coverage probabilities (as low as 0.0623 for $n = 20$).

The other symmetric distributions studied herein (T_5, T_7, T_{10}, and Laplace) produce coverage probabilities around 90% for medium kurtosis, but are somewhat too conservative (coverage probabilities above 90%) for high kurtosis, and are somewhat anti-conservative (coverage probabilities below 90%) for low kurtosis. However, when the coverage probabilities are quite low (e.g., 0.7659 for Laplace distribution with $n = 20$) and the kurtosis is low, the likelihood of achieving low kurtosis is also quite low (e.g., 0.0055).

Table 1. Skewness coverage probabilities

Distribution	Sample size	Proportion high skewness	Coverage prob low skewness	Coverage prob high skewness	Coverage probability overall
.5	5	0.1000	0.9001	0.8996	0.9000
Normal	10	0.1000	0.9001	0.9000	0.9001
	15	0.1000	0.9001	0.9002	0.9001
	20	0.1000	0.9000	0.8998	0.9000
.5	5	0.0937	0.8936	0.8375	0.8884
Uniform	10	0.0467	0.9175	0.5175	0.8988
	15	0.0297	0.9174	0.3284	0.8999
	20	0.0220	0.9148	0.2446	0.9000
.5	5	0.1354	0.9055	0.9298	0.9088
T_5	10	0.2043	0.8991	0.9240	0.9042
	15	0.2503	0.8992	0.9128	0.9026
	20	0.2850	0.9004	0.9051	0.9017
.5	5	0.1223	0.9040	0.9209	0.9060
T_7	10	0.1690	0.8991	0.9194	0.9025
	15	0.2004	0.8988	0.9118	0.9014
	20	0.2244	0.8993	0.9063	0.9009
.5	5	0.1141	0.9028	0.9137	0.9040
T_{10}	10	0.1448	0.8993	0.9151	0.9016
	15	0.1656	0.8988	0.9106	0.9008
	20	0.1813	0.8990	0.9070	0.9005
.5	5	0.1652	0.9101	0.9537	0.9173
Laplace	10	0.2601	0.8995	0.9228	0.9056
	15	0.3130	0.9034	0.9008	0.9026
	20	0.3480	0.9074	0.8902	0.9014
.5	5	0.3668	0.7456	0.8280	0.7758
χ^2_1	10	0.6777	0.7559	0.8436	0.8153
	15	0.8619	0.7476	0.8495	0.8355
	20	0.9460	0.7366	0.8538	0.8475
.5	5	0.2458	0.8130	0.8865	0.8311
χ^2_2	10	0.4848	0.8157	0.8929	0.8531
	15	0.6780	0.8110	0.8899	0.8645
	20	0.8090	0.8052	0.8870	0.8714
.5	5	0.1978	0.8402	0.9020	0.8524
χ^2_3	10	0.3848	0.8405	0.9104	0.8674
	15	0.5556	0.8373	0.9059	0.8754
	20	0.6899	0.8335	0.9014	0.8803
	5	0.2458	0.8131	0.8865	0.8312
Exponential	10	0.4848	0.8157	0.8927	0.8530
	15	0.6781	0.8110	0.8899	0.8645
	20	0.8089	0.8055	0.8868	0.8713

The χ^2_2, χ^2_3, and Exponential distributions tend to achieve the nominal coverage probability of 90% when the dataset produces high kurtosis, but achieves low coverage (between 80% and 87%) when the dataset produces low or medium kurtosis. The coverage probabilities for the χ^2_1 distribution are somewhat low; around 74% for datasets producing low kurtosis, 80% for medium kurtosis, and 87% for high kurtosis. The likelihood of a dataset producing low kurtosis is much lower than from a dataset producing high kurtosis, for these four skewed distributions.

Table 2. Kurtosis coverage probabilities

Distribution	Sample size	Proportion low kurtosis	Proportion high kurtosis	Coverage prob low kurtosis	Coverage prob middle kurtosis	Coverage prob high kurtosis
.5	5	0.0501	0.0500	0.9001	0.9000	0.8999
Normal	10	0.0500	0.0501	0.9003	0.9000	0.9003
	15	0.0500	0.0501	0.8997	0.9000	0.9004
	20	0.0500	0.0500	0.9001	0.9000	0.8999
.5	5	0.0964	0.0482	0.9241	0.8878	0.8285
Uniform	10	0.1923	0.0116	0.9577	0.8925	0.3606
	15	0.3230	0.0026	0.9666	0.8707	0.1418
	20	0.4595	0.0006	0.9663	0.8447	0.0623
.5	5	0.0419	0.0731	0.8916	0.9077	0.9340
T_5	10	0.0301	0.1384	0.8720	0.9000	0.9369
	15	0.0228	0.1922	0.8676	0.8970	0.9288
	20	0.0177	0.2406	0.8673	0.8957	0.9227
.5	5	0.0441	0.0638	0.8940	0.9054	0.9238
T_7	10	0.0348	0.1065	0.8807	0.9001	0.9291
	15	0.0287	0.1418	0.8777	0.8982	0.9244
	20	0.0240	0.1734	0.8770	0.8974	0.9201
.5	5	0.0457	0.0585	0.8959	0.9037	0.9160
T_{10}	10	0.0389	0.0857	0.8871	0.9001	0.9226
	15	0.0340	0.1080	0.8847	0.8990	0.9205
	20	0.0300	0.1281	0.8847	0.8984	0.9173
.5	5	0.0370	0.0938	0.8688	0.9149	0.9598
Laplace	10	0.0173	0.1902	0.7629	0.8984	0.9485
	15	0.0095	0.2626	0.7587	0.8926	0.9358
	20	0.0055	0.3283	0.7659	0.8896	0.9277
.5	5	0.0766	0.2582	0.7607	0.7527	0.8389
χ_1^2	10	0.0390	0.3832	0.7406	0.7917	0.8585
	15	0.0210	0.5192	0.7354	0.8025	0.8685
	20	0.0111	0.6234	0.7364	0.8073	0.8729
.5	5	0.0680	0.1543	0.8332	0.8178	0.8971
χ_2^2	10	0.0536	0.2534	0.8145	0.8380	0.9026
	15	0.0407	0.3459	0.8069	0.8460	0.9044
	20	0.0291	0.4265	0.8033	0.8493	0.9040
.5	5	0.0624	0.1173	0.8582	0.8435	0.9114
χ_3^2	10	0.0563	0.1969	0.8452	0.8560	0.9173
	15	0.0486	0.2672	0.8379	0.8621	0.9166
	20	0.0396	0.3309	0.8338	0.8650	0.9148
	5	0.0680	0.1543	0.8331	0.8180	0.8971
Exponential	10	0.0536	0.2533	0.8144	0.8379	0.9026
	15	0.0407	0.3459	0.8074	0.8459	0.9043
	20	0.0291	0.4266	0.8036	0.8494	0.9041

Due to these lower coverage probabilities for these four skewed distributions when kurtosis is low, the recommendation herein to is avoid using the t-test when the dataset produces a low kurtosis. The drawback is that the Uniform distribution frequently produces low kurtosis, but the Uniform distribution tends to perform well under nonparametric tests, such as the Wilcoxon Signed-Rank Test.

5 Conclusions

By examining the tables, we analyze the performance of the t-test for several distributions and determine when the use of the test is appropriate. Considering each distribution, there is a pattern of slightly higher coverage probabilities for datasets that meet requirements for high skewness as opposed to datasets with low skewness. When looking at kurtosis, we can also see a pattern of slightly higher coverage probabilities for most datasets with high kurtosis as opposed to datasets with low or medium kurtosis. An exception to these results occurs with the Uniform distribution, where the coverage probabilities are extremely low for datasets with high skewness or high kurtosis, especially when the sample sizes are larger. These results provide evidence that students and researchers may be able to relax assumptions typically required for using the t-test. Statisticians could be less wary of using the t-test and t-confidence intervals if skewness or kurtosis is high. However, if kurtosis is low, the t-procedures are not ideal. While alternatives to the t-test, such as the Wilcoxon Signed-Rank Test, may be preferred for Uniform distributions, the t-test should not necessarily be avoided when using small datasets which have high skewness or high kurtosis. Future research involves generalization to the multivariate Hotelling's T^2 test.

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Competing Interests

Authors have declared that no competing interests exist.

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