



Spectral Adomian Decomposition Method for the MHD Three-Dimensional Flow due to a Stretching Sheet in a Porous Medium

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Authors' contributions

This work was carried out in collaboration among all authors. Author YD and Author MA managed and analyses the results of the study and they also managed the literature searches. Author AK designed the problem formulation and analysis, wrote the protocol, and wrote the first draft of the manuscript. All authors read and approved the final manuscript.

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Abstract

An approximate solution is obtained of the steady, laminar three-dimensional flow for an incompressible, viscous fluid past a stretching sheet using the Spectral Adomian Decomposition Method (SADM). The governing partial differential equations are transformed into ordinary differential equations using suitable transformations. A comparison between the obtained results with solutions obtained early in the literature and the numerical solution has been made to test the validity, accuracy and convergence of the SADM. The effects of physical parameters on the velocity are determined and discussed.

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1 Introduction

There has been growing interest in Magneto-hydrodynamics (MHD) flow through porous medium due to the fact that fluid metals are electrically conducting and possess thermal properties also. Recently, this type of the fluid flow has received attention of many researchers due to its applications in technological models such as MHD generator and plasma studies. In view of its importance in polymer industry, the flow due to a stretching sheet has been received attention and extensively studied since [1] gave the closed form solution for the two-dimensional flow. The effect of heat transfer on the two-dimensional flow due to stretching of a sheet have been studied by several researchers (see [2, 3, 4, 5, 6]). [7] studied the more general three-dimensional flow. He derived a perturbation solution around the two-dimensional flow, and he used this derivation as a guideline for obtaining the numerical solution of the fully generalized problem. He mention a related axisymmetric exact solution of the Navier-Stokes equations. Besides [7], the solution of the generalized three-dimensional flow past a stretching sheet has been discussed by [8], he used the Ackroyd method [9] of an infinite series of negative exponentials. [10] applied the technique of [11] which generates the solution non-iteratively. [12, 13] studied nanoparticle involving solar radiation and solar collector with turbulator involving nanomaterial turbulent regime.

In the present work we extend the steady, laminar three-dimensional flow of a viscous, incompressible fluid due to a stretching sheet in a porous media considering the MHD on the flow and attempt to obtain its solution using the Spectral Adomian Decomposition Method (SADM). The SADM method can be used in place of traditional numerical methods such as finite differences, Runge-Kutta shooting methods, finite elements in solving non-linear boundary value problems.

2 Mathematical Formulation

Consider a three-dimensional boundary layer flow of an incompressible elasto-viscous fluid over a stretching sheet in porous medium. We consider a magnetic field which is not considered for small magnetic Reynolds number. Under the above assumptions, the governing equations for this problem can be written as follows [7]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \nu \frac{\partial^2 u}{\partial z^2} - \frac{\nu}{k^*} u - \frac{\sigma B_0^2}{\rho} u, \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \nu \frac{\partial^2 v}{\partial z^2} - \frac{\nu}{k^*} v - \frac{\sigma B_0^2}{\rho} v, \quad (3)$$

in which (u, v, w) denote the velocity components in the (x, y, z) directions, respectively, k^* is the permeability of the porous medium, σ is the electrical conductivity, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, B_0^2 is the uniform magnetic field, and ρ is the density of the fluid. The boundary conditions are taken as follows:

$$\left. \begin{aligned} u = u_w(x) = ax, \quad v = v_w(y) = by, \quad w = 0, \quad \text{at} \quad z = 0, \\ u \rightarrow 0, \quad v \rightarrow 0, \quad \text{as} \quad z \rightarrow \infty, \end{aligned} \right\} \quad (4)$$

where a and b are the constants of proportionality. Using the similarity variables

$$u = axf'(\eta), \quad v = ayg'(\eta), \quad w = -\sqrt{a\nu}(f(\eta) + g(\eta)), \quad \eta = z\sqrt{\frac{a}{\nu}}, \quad (5)$$

the continuity equation (1) is automatically satisfied and the system of partial differential equations (2) and (3) is converted into ordinary differential equations

$$f''' + (f + g)f'' - f'^2 - Kf' - M^2f' = 0, \tag{6}$$

$$g''' + (f + g)g'' - g'^2 - Kg' - M^2g' = 0, \tag{7}$$

where the prime symbol represents the derivative with respect to η and $K = \frac{\nu}{k^*a}$ is the permeability parameter, $M^2 = \frac{\sigma B_0^2}{a\rho}$ is the Hartman number. The corresponding boundary conditions are

$$\left. \begin{aligned} f(0) + g(0) = 0, & \quad f'(0) = 1, & \quad g'(0) = \beta, \\ f'(\eta) \rightarrow 0, & \quad g'(\eta) \rightarrow 0 & \quad \text{as } \eta \rightarrow \infty, \end{aligned} \right\} \tag{8}$$

where $\beta = \frac{b}{a}$ is a ratio parameter.

3 Fundamentals of Adomian Decomposition Method (ADM)

In this section, the review of the standard Adomian decomposition method [14, 15, 16, 17] is presented. We start by considering the following differential equation

$$Lu(x) + Ru(x) + Nu(x) = g(x); \tag{9}$$

where L is the highest-order derivative which is assumed to be invertible, R is a linear differential operator of less order than L , Nu represents the nonlinear terms, and $g(x)$ is known analytic function. The method is based on applying the inverse operator L^{-1} formally to the expression

$$Lu(x) = g(x) - Ru(x) - N(u(x)). \tag{10}$$

So, by using the given conditions we obtain

$$u(x) = f(x) - L^{-1}(Ru(x)) - L^{-1}(N(u(x))), \tag{11}$$

where the function $f(x)$ represents the terms arising from integrating the source term $g(x)$ and from using the given conditions, all are assumed to be prescribed. The standard Adomian decomposition method defines the solution $u(x)$ by the series

$$u(x) = \sum_{i=0}^{\infty} u_i(x), \tag{12}$$

where the components u_0, u_1, u_2, \dots , are usually determined recursively by using the relation

$$u_{k+1} = -L^{-1}(Ru_k) - L^{-1}(Nu_k), \quad k \geq 0. \tag{13}$$

It is important to note that the decomposition method suggests that the zeroth component u_0 is usually identified by the function f described above. For nonlinear equations, the nonlinear operator $Nu = F(u)$ is usually represented by an infinite series of the so-called Adomian polynomials

$$F(u) = \sum_{k=0}^{\infty} A_k, \tag{14}$$

where Adomian polynomials A_n may be computed by the formula

$$A_n = \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} N \left(\sum_{i=0}^n \lambda^i u_i \right) \right]_{\lambda=0}. \tag{15}$$

4 Spectral Adomian Decomposition Method (SADM)

In this section, we will apply the SADM to solve the system of equations (6) and (7) together with the boundary conditions (8). The technique is based on the blending of the Chebyshev pseudospectral methods and the Adomian decomposition method. The physical region $[0, \infty]$ is transformed to the region $[-1, 1]$ by using the mapping

$$\frac{\eta}{L} = \frac{x+1}{2}, \quad -1 \leq x \leq 1. \quad (16)$$

We discretize the domain $[-1, 1]$ using the Gauss-Lobatto collocation points given by

$$x = \cos \frac{\pi j}{N}, \quad j = 0, 1, 2, \dots, N, \quad (17)$$

where N is the number of collocation points used. It is also convenient to make the boundary conditions homogeneous by making use of the transformations

$$f(\eta) = f_i(\eta) + \sum_{m=0}^{i-1} f_m(\eta), \quad g(\eta) = g_i(\eta) + \sum_{m=0}^{i-1} g_m(\eta), \quad (18)$$

where $f_m(\eta)$ and $g_m(\eta)$ are chosen so as to satisfy boundary conditions (8). Substituting equations (16) and (18) in equations (6) and (7) gives

$$\begin{aligned} \frac{8}{L^3} f_i''' + \frac{4}{L^2} (f_m + g_m) f_i'' - \frac{2}{L} (2f_m' + K + M^2) f_i' + f_m'' f_i + f_m' g_i \\ + \frac{4}{L^2} (f_i f_i'' + g_i f_i'' - f_i'^2) = R_1, \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{8}{L^3} g_i''' + \frac{4}{L^2} (f_m + g_m) g_i'' - \frac{2}{L} (2g_m' + K + M^2) g_i' + g_m'' g_i + g_m' f_i \\ + \frac{4}{L^2} (f_i g_i'' + g_i g_i'' - g_i'^2) = R_2, \end{aligned} \quad (20)$$

subject to the boundary conditions

$$f_i(-1) = f_i'(-1) = f_i'(1) = g_i(-1) = g_i'(-1) = g_i'(1) = 0, \quad (21)$$

where

$$R_1 = M^2 f_m' + K f_m' - f_m f_m'' + f_m'^2 - f_m' g_m - f_m'', \quad (22)$$

$$R_2 = M^2 g_m' + K g_m' - f_m g_m'' - g_m g_m'' + g_m'^2 - g_m'', \quad (23)$$

The initial approximations $f_0(x)$ and $g_0(x)$ for the SADM solution of equations (19) and (20) are obtained by solving the linear part of (19) and (20), namely

$$\frac{8}{L^3} f_i''' + \frac{4}{L^2} (f_m + g_m) f_i'' - \frac{2}{L} (2f_m' + K + M^2) f_i' + f_m'' f_i + f_m' g_i = R_1, \quad (24)$$

$$\frac{8}{L^3} g_i''' + \frac{4}{L^2} (f_m + g_m) g_i'' - \frac{2}{L} (2g_m' + K + M^2) g_i' + g_m'' g_i + g_m' f_i = R_2, \quad (25)$$

subject to the boundary conditions

$$f_0(-1) = f_0'(-1) = f_0'(1) = g_0(-1) = g_0'(-1) = g_0'(1) = 0. \quad (26)$$

The system of equations (24) and (25) is solved using the Chebyshev pseudospectral method where the unknown functions $f_0(x)$ and $g_0(x)$ are approximated as truncated series of Chebyshev polynomials of the form

$$f_i(x) \approx \sum_{k=0}^N f_i(x_k) T_k(x_j), \quad g_i(x) \approx \sum_{k=0}^N g_i(x_k) T_k(x_j), \quad j = 0, 1, \dots, N, \quad (27)$$

where T_k is the k^{th} Chebyshev polynomial given by

$$T_k(x) = \cos [k \cos^{-1}(x)]. \quad (28)$$

The derivatives of the variables at the collocation points are represented as

$$\frac{d^r f_i}{dx^r} = \sum_{k=0}^N \mathcal{D}_{kj}^r f_i(x_k), \quad \frac{d^r g_i}{dx^r} = \sum_{k=0}^N \mathcal{D}_{kj}^r g_i(x_k), \quad j = 0, 1, \dots, N, \quad (29)$$

where r is the order of differentiation and \mathcal{D} being the Chebyshev spectral differentiation matrix whose entries are defined as ([18, 19, 20])

$$\left. \begin{aligned} \mathcal{D}_{00} &= \frac{2N^2 + 1}{6}, \\ \mathcal{D}_{jk} &= \frac{c_j (-1)^{j+k}}{c_k x_j - x_k}, \quad j \neq k; \quad j, k = 0, 1, \dots, N, \\ \mathcal{D}_{kk} &= -\frac{2(1 - x_k^2)}{2N^2 + 1}, \quad k = 1, 2, \dots, N - 1, \\ \mathcal{D}_{NN} &= -\frac{2N^2 + 1}{6}, \end{aligned} \right\} \quad (30)$$

where $c_0 = c_N = 2$ and $c_j = 1$ with $1 \leq j \leq N - 1$. Substituting equations (27), (28), (29), (30) and (17) into equations (24) and (25) leads to the matrix equation

$$\mathbf{A}\mathbf{X}_0 = \mathbf{R}_0. \quad (31)$$

In equation (31), \mathbf{A} is a $(2N + 2) \times (2N + 2)$ square matrix and \mathbf{X}_0 and \mathbf{R}_0 are $(2N + 2) \times 1$ column vectors defined by

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad \mathbf{X}_0 = \begin{bmatrix} f_0 \\ g_0 \end{bmatrix}, \quad \mathbf{R}_0 = \begin{bmatrix} \mathbf{r}_{01} \\ \mathbf{r}_{02} \end{bmatrix}, \quad (32)$$

with

$$\begin{aligned} A_{11} &= \frac{8}{L^3} \mathcal{D}^3 + \frac{4}{L^2} ([f_m] + [g_m]) \mathcal{D}^2 - \frac{2}{L} (2[f'_m] + K + M^2) \mathcal{D} + [f''_m], \\ A_{12} &= [f''_m], \\ A_{21} &= [g''_m], \\ A_{22} &= \frac{8}{L^3} \mathcal{D}^3 + \frac{4}{L^2} ([f_m] + [g_m]) \mathcal{D}^2 - \frac{2}{L} (2[g'_m] + K + M^2) \mathcal{D} + [g''_m], \\ \mathbf{r}_{01} &= [R_1(x_0), R_1(x_1), \dots, R_1(x_{N-1}), R_1(x_N)]^T, \\ \mathbf{r}_{02} &= [R_2(x_0), R_2(x_1), \dots, R_2(x_{N-1}), R_2(x_N)]^T, \end{aligned}$$

where T stands for transpose and $[\cdot]$ is a diagonal matrix of size $(N + 1) \times (N + 1)$. After modifying the matrix system (31) to incorporate boundary conditions (26), the initial approximation solution of (19) and (20) is obtained as

$$\mathbf{X}_0 = \mathbf{A}^{-1} \mathbf{R}_0. \quad (33)$$

For the higher order approximations for the SADM solution of (19) and (20), one can write equations (19) and (20) using equation (10) as follows

$$\begin{aligned} A_{11} f_{k+1} + A_{12} g_{k+1} &= -\frac{4}{L^2} \sum_{i=0}^k (f_i (D^2 f_{k-i}) + g_i (D^2 f_{k-i}) - (D f_{k-i})^2) = r_{k+1,1}, \quad k = 0, 1, 2, \dots \\ A_{21} f_{k+1} + A_{22} g_{k+1} &= -\frac{4}{L^2} \sum_{i=0}^k (f_i (D^2 g_{k-i}) + g_i (D^2 g_{k-i}) - (D g_{k-i})^2) = r_{k+1,2}, \quad k = 0, 1, 2, \dots \end{aligned}$$

The above system of equations can be written as

$$\mathbf{A}\mathbf{X}_{k+1} = \mathbf{R}_{k+1}, \tag{34}$$

where

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad \mathbf{X}_0 = \begin{bmatrix} f_{k+1} \\ g_{k+1} \end{bmatrix}, \quad \mathbf{R}_{k+1} = \begin{bmatrix} \mathbf{r}_{k+1,1} \\ \mathbf{r}_{k+1,2} \end{bmatrix}, \tag{35}$$

subject to the boundary conditions

$$f_{k+1}(-1) = f'_{k+1}(-1) = f'_{k+1}(1) = g_{k+1}(-1) = g'_{k+1}(-1) = g'_{k+1}(1) = 0. \tag{36}$$

After modifying the matrix system (34) to incorporate boundary conditions (36), the solution is obtained as

$$\mathbf{X}_{k+1} = \mathbf{A}^{-1}\mathbf{R}_{k+1}. \tag{37}$$

Thus, starting from the first approximation f_0 and g_0 , the higher-order approximation f_{k+1} and g_{k+1} can be obtained through recursive formula (37). Now the final solution is given by

$$f(\eta) = f_m(\eta) + \sum_{i=0}^{\infty} f_i(\eta), \quad g(\eta) = g_m(\eta) + \sum_{i=0}^{\infty} g_i(\eta). \tag{38}$$

5 Results and Discussion

In this section we give the Spectral Adomian Decomposition method results for the main parameters affecting the flow. The accuracy and efficiency of the solutions are demonstrated by comparing the current results against the numerical solutions obtained using the MATLAB routine `bvp4c`. In generating the presented results it was determined through numerical experimentation that $\infty \simeq L = 15$ and $N = 120$ gave sufficient accuracy for the SADM.

In Table 1, the SADM results were compared with those reported by [7] and `bvp4c` numerical results for $-f''(0)$ and $-g''(0)$ for different values of β when $M = 0$, $K = 0$. It can be seen from the table that the SADM method gives much more accurate results than those obtained by [7].

Table 1. Comparison between the SADM, [7] and numerical results for $-f''(0)$ and $-g''(0)$ for different values of β when $M = 0$, $K = 0$

β	$-f''(0)$			$-g''(0)$		
	SADM	[7]	Numeric	SADM	[7]	Numeric
0	1	1	1	0	0	0
0.25	1.048811	1.048813	1.048811	0.194564	0.194564	0.194564
0.5	1.093095	1.093097	1.093095	0.465205	0.465205	0.465205
0.75	1.134486	1.134485	1.134486	0.794618	0.794622	0.794618
1.	1.173721	1.173720	1.173721	1.173721	1.117372	1.173721

Tables 2 and 3 showed a comparison between the SADM and the numerical results of $-f''(0)$ and $-g''(0)$ at various values of K and M for different iterations of the SADM procedure. Convergence of the SADM results to the numerical approximation is achieved starting from the seventh iteration for six decimal places.

Table 2. SADM results against numerical solution of $-f''(0)$ at different values of K and M when $\beta = 0.5$.

K	M	4 th iteration	5 th iteration	6 th iteration	7 th iteration	Numeric
	0.1	1.141109	1.141116	1.141118	1.141118	1.141119
0.1	0.5	1.240024	1.240033	1.240036	1.240037	1.240037
	1.0	1.510012	1.510021	1.510023	1.510024	1.510024
	0.1	1.302091	1.302100	1.302103	1.302104	1.302104
0.5	0.5	1.390272	1.390282	1.390284	1.390285	1.390285
	1.0	1.636448	1.636455	1.636457	1.636457	1.636457
	0.1	1.480117	1.480126	1.480128	1.480129	1.480129
1	0.5	1.558599	1.558607	1.558609	1.558609	1.558609
	1.0	1.782151	1.782156	1.782157	1.782157	1.782157

Table 3. SADM results against numerical solution of $-g''(0)$ at different values of K and M when $\beta = 0.5$

K	M	1 st iteration	2 nd iteration	3 rd iteration	4 th iteration	Numeric
	0.1	0.492892	0.492981	0.492997	0.493000	0.493001
0.1	0.5	0.549045	0.549207	0.549233	0.549238	0.549240
	1.0	0.697498	0.697780	0.697811	0.697815	0.697815
	0.1	0.583709	0.583911	0.583941	0.583946	0.583948
0.5	0.5	0.632354	0.632600	0.632633	0.632637	0.632638
	1.0	0.765400	0.765695	0.765721	0.765722	0.765722
	0.1	0.681319	0.681594	0.681626	0.681630	0.681630
1	0.5	0.723687	0.723976	0.724005	0.724008	0.724008
	1.0	0.842786	0.843073	0.843091	0.843091	0.843091

Table 4. Absolute error norms values of $f(\eta)$ and $g(\eta)$ when $M = 0.5, K = 0.5, \beta = 0.1$ at different order of iteration N

N	SADM	
	$f(\eta)$	$g(\eta)$
8	$1.41043e - 7$	$4.28632e - 9$
9	$9.00603e - 8$	$4.46446e - 9$
10	$8.51249e - 8$	$4.58746e - 9$
12	$8.42685e - 8$	$4.62706e - 9$
20	$8.42241e - 8$	$4.62931e - 9$

In Table 4, we give the maximum absolute errors between the SADM and numerical solutions of $f(\eta)$ and $g(\eta)$ for varied values of collocation points N . It is clear from the Table, the SADM results are very accurate to the numerical solution because the errors are very small at the collocation points.

Figs 1 and 2 showed the influence of the various physical parameters M and K on $f'(\eta), g'(\eta)$ and a comparison between the SLM and numerical results. In Fig. 1. we plotted the effect of various values of the magnetic field M at fixed values of K . It is noted that M reduces the boundary layer thickness. Fig. 2. represents the the effect of the permeability parameter K on dimensionless velocity distributions $f'(\eta)$ and $g'(\eta)$ when M is fixed. It shows that the velocities reduced as K increased.

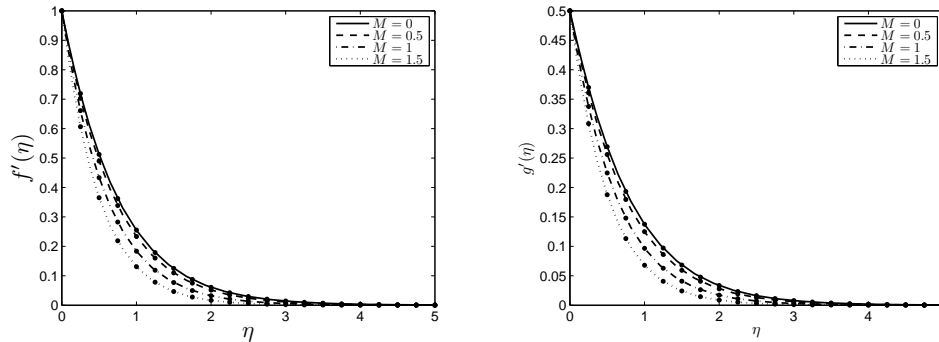


Fig. 1. Comparison of the numerical results (circles) and SLM approximation for $f'(\eta)$ and $g'(\eta)$ for different values of M when $K = \beta = 0.5$

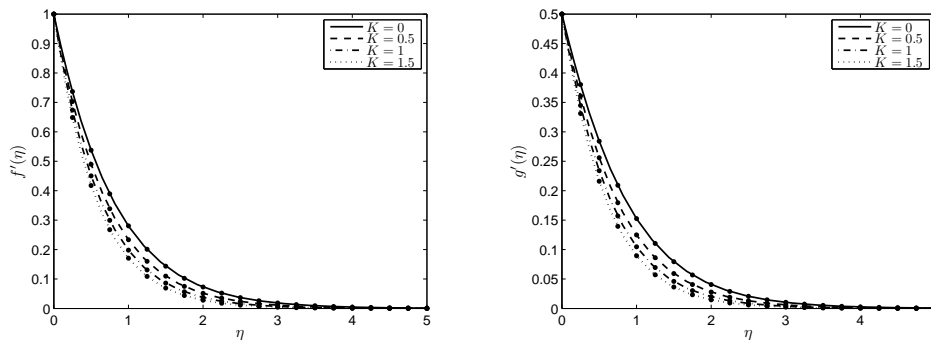


Fig. 2. Comparison of the numerical results (circles) and SLM approximation for $f'(\eta)$ and $g'(\eta)$ for different values of K when $M = \beta = 0.5$

6 Conclusion

In this work we have investigated the MHD flow due to a stretching sheet in a porous medium. A similarity transformation reduced the governing partial differential equations into ordinary differential equations which were then solved using the SADM. The SADM procedure transforms the differential equations into a system of algebraic equations which is easier and faster to solve.

We found high convergence of the series solution and we have shown that the SADM gives good accuracy and computational efficiency. The results indicate that an increases in the magnetic and permeability parameters reduces the velocities profiles.

Competing Interests

Authors have declared that no competing interests exist.

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