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Couple Stress Fluid Past a Porous Spheroidal Shell with Solid Core under Stokesian Approximation

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Authors' contributions

This work was carried out in collaboration between both authors. Authors TKVI and TSLR jointly formulated the problem. Author TKVI indicated the procedure to solve it and author TSLR obtained the solutions. Further the programming and numerical computations were carried out by author TSLR. The two authors jointly interpreted the results and the final draft was prepared jointly. Both authors read and approved the final manuscript and they are responsible for the results.

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ABSTRACT

The present paper deals with the flow of an incompressible couple stress fluid past a porous spheroidal shell which is made up of two confocal prolate spheroids S_0 and S_1 where S_0 is within S_1. The spheroid S_0 is taken to be a solid and the annular region between S_0 and S_1 is porous, with the boundary of S_1 being pervious. The flow outside S_1 is governed by the linearized version of Stokes' couple stress fluid flow equations and that within the porous region is governed by the classical Darcy's law. The resulting equations are then solved analytically for the velocity and pressure fields and drag experienced by the body is obtained. The variation of drag with the different parameters like the material and the geometric is studied numerically and the results are presented through graphs. Stream lines are also plotted to understand the flow pattern.

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1. INTRODUCTION

Joseph and Tao [1], as early as in 1964 made a study regarding the effect of permeability on the slow motion of a porous sphere in a viscous liquid. Since then, a number of papers appeared dealing with flow of a viscous liquid past and within porous bodies using analytical or numerical methods in view of their importance in geophysical, industrial and engineering applications. To cite a few: Padmavathi and Amaranth obtained a solution for the problem of Stokes flow of a viscous fluid past a porous sphere [2]. Qin and Kaloni obtained a solution for the creeping flow past a porous spherical shell [3]. Zlatanovski dealt with the analytical solution to the problem of axisymmetric creeping flow of a viscous liquid past a porous prolate spheroidal particle [4]. Vainshtein et al. [5] in cited a number of references dealing with flow past and within porous spheres with the flows outside the body governed by Navier Stokes equations and within the porous region governed by classical Darcian model or Brinkman model. These authors have made a significant contribution by dealing with the flow of a viscous liquid past and within a permeable spheroid [5]. Srinivasacharya studied the flow of an incompressible viscous fluid within an axially symmetric porous approximate shell [6].

To the extent the authors have surveyed, the above important problems have not been studied with reference to polar fluid models which are more general than the classical Newtonian viscous fluid model. While there exist several polar fluid models, Eringen's micropolar fluid model [7] and Stokes' couple stress fluid model [8] are two significant generalizations to the classical model. These two arise from different stand points. The micropolar fluid model of Eringen takes into account both the mechanical interactions as well as the micro structure of the molecules within a fluid volume element. The couple stress fluid model of Stokes takes into consideration the mechanical interactions taking place across a surface in the fluid medium and is not concerned with the micro structure. A number of axisymmetric Stokes flow problems related to these fluids dealing with solid sphere, spheroid and approximate sphere can be seen in the works of Lakshmana Rao et al. [9-15], Ramkissoon and Majumdar [16], Ramkisson [17] and Iyengar et al. [18,19]. However, Srinivasacharya et al. [20] considered the creeping flow of a micropolar fluid past a porous sphere and Ramana Murthy et al. [21] studied this problem with respect to a couple stress fluid. Recently, the authors also studied the problem of flow of couple stress fluid past a porous spheroidal shell with liquid core [22].

In this paper, we investigate the slow steady flow of an incompressible couple stress fluid past a porous spheroidal shell containing a rigid confocal spheroidal core. The region outside the outer spheroid is occupied by an incompressible couple stress fluid and the flow in the porous region is governed by the classical Darcy's law. As an initial trial for solving the differential equations governing the flow, which are of higher order than the Navier Stokes equations, we have solved this problem with a certain set of boundary conditions (to be explained later) and obtained the velocity components in terms of Legendre functions, Associated Legendre functions, prolate radial and angular wave functions. The drag on the spheroid is evaluated for diverse values of the couple stress parameter, permeability parameter and the size of the spheroid. The stream lines are plotted for diverse values of these parameters as well. It is noticed that an increase in the couple stress viscosity decreases the drag significantly and a decrease in the couple stress viscosity and a simultaneous increase in the permeability parameter has a disturbing effect on the flow pattern.

2. BASIC EQUATIONS

Couple stress fluid model given by V.K. Stokes [8,23] is based on the presumption that the fluent medium can sustain couple stresses. Here the non-symmetric stress tensor t_{ij} and the couple stress tensor m_{ij} are given by

$$t_{ij} = -p\delta_{ij} + \lambda div\overline{q}\delta_{ij} + 2\mu d_{ij} - \frac{1}{2}\varepsilon_{ijk} \left\{ m_{,k} + 4\eta_1\omega_{k,rr} + \rho C_k \right\} \quad (1)$$

$$m_{ij} = \frac{1}{3}m\delta_{ij} + 4\eta_1\omega_{j,i} + 4\eta_1'\omega_{i,j} \quad (2)$$

The notation is as in [22]. These material constants are constrained by the inequalities

$$\mu \geq 0 \ , 3\lambda + 2\mu \geq 0 \ , \ \eta_1 \geq 0 \ , \ |\eta_1'| \leq \eta_1 \qquad (3)$$

The parameter $\sqrt{\dfrac{\eta_1}{\mu}}$ is a characteristic measure of the polarity of the fluid model which is zero in the case of nonpolar fluid.

The equations of motion concerning couple stress fluid flow are

$$\frac{\partial \rho}{\partial t} + \rho div(\vec{q}) = 0 \qquad (4)$$

$$\rho \frac{d\vec{q}}{dt} = \rho \vec{f} + \frac{1}{2} curl(\rho \, \vec{c}) + div\tau^{(s)} + \frac{1}{2} curl(divM) \qquad (5)$$

where ρ is the density of the fluid, $\tau^{(s)}$ is the symmetric part of the force stress diad and M is the couple stress diad and $\vec{f}$, $\bar{c}$ are the body force per unit mass and body couple per unit mass respectively.

3. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider two confocal prolate spheroids S_0 and S_1 with foci P,Q, where PQ=2c units. Let O be the midpoint of PQ. Introduce the cylindrical polar coordinate system (r,θ,z) with respect to O as origin and OQ extended on either side as Z axis.

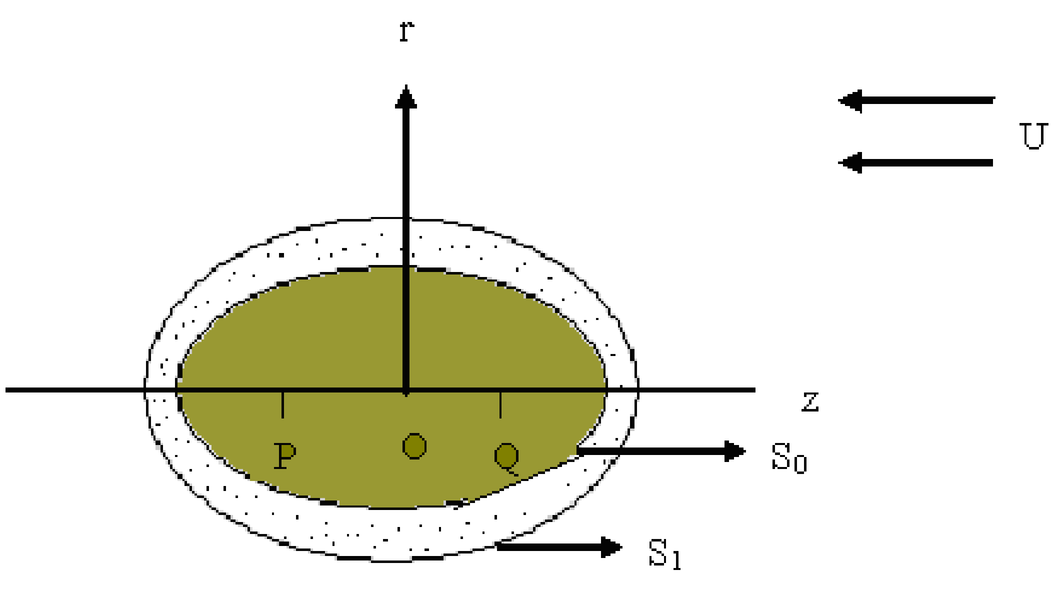
Let us consider the uniform slow stationary flow of an incompressible couple stress fluid past the spheroid S_1 with velocity U in the direction of the z-axis far away from the body. Let us denote the region outside the spheroid S_1 by F_1 and the porous region between S_0 and S_1 by F_0. The boundary of S_0 is impervious while that of S_1 is pervious.

We assume that the flow in the region F_1 is governed by the incompressible couple stress fluid flow equations and the fluid flow within F_0 is governed by the classical Darcy's law. Since the flow is slow, we assume that the flow is axisymmetric and is the same in any meridian plane and thus the flow variables are independent of the azimuth angle.

We shall introduce the prolate spheroidal coordinates (ξ, η, ϕ) with $(\bar{e}_\xi, \bar{e}_\eta, \bar{e}_\phi)$ as base vectors and (h_1, h_2, h_3) as the corresponding scale factors through the definition

$$z + ir = c \cosh (\xi + i\eta) \qquad (6)$$

We assume that the flow is Stokesian as in the classical investigation of the problem by Payne and Pell in the case of classical viscous fluid [24] and Lakshmana Rao and Iyengar in the case of micropolar fluid [11]. This enables us to drop the inertial terms in the momentum equation.



Let ($\overline{q}^{(1)}, p^{(1)}$) denote the velocity and pressure in the region F_1 and let ($\overline{q}^{(0)}, p^{(0)}$) be the velocity and pressure in the porous region F_0. In view of the symmetry of the flow, we take

$$\overline{q}^{(1)} = \mathrm{u}^{(1)}(\xi,\eta)\overline{e}_{\xi} + \mathrm{v}^{(1)}(\xi,\eta)\overline{e}_{\eta} \tag{7}$$

$$p^{(1)} = \mathrm{p}^{(1)}(\xi,\eta) \tag{8}$$

The basic equations governing the steady Stokesian flow in region F_1 can be written in the form

$$div(\vec{q}^{(1)}) = 0 \tag{9}$$

$$\begin{aligned} &grad\ p^{(1)} + \mu\ \mathrm{curl}\ curl\ \overline{q}^{(1)} + \\ &\eta_1 curl\ curl\ curl\ curl\ \vec{q}^{(1)} = 0 \end{aligned} \tag{10}$$

In view of the continuity equation, we introduce the stream function $\psi^{(1)}$ through

$$h_2 h_3 u^{(1)} = -\frac{\partial \psi^{(1)}}{\partial \eta};\ h_1 h_3 v^{(1)} = \frac{\partial \psi^{(1)}}{\partial \xi} \tag{11}$$

Using (7) and (11)

$$curl\vec{q}^{(1)} = \left(\frac{1}{h_3} E^2 \psi^{(1)}\right)\vec{e}_{\phi} \tag{12}$$

in which the Stokes stream function operator E^2 is given by

$$E^2 = \frac{h_3}{h_1 h_2}\left(\frac{\partial}{\partial \xi}\left(\frac{h_2}{h_1 h_3}\frac{\partial}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(\frac{h_1}{h_2 h_3}\frac{\partial}{\partial \eta}\right)\right) \tag{13}$$

Evaluating the expressions for $\mathrm{curl}\ curl\ \vec{q}^{(1)}$, $curl\ curl\ curl\ curl\ \vec{q}^{(1)}$, the basic equations describing the flow in the region F_1 is given by

$$\frac{\partial p^{(1)}}{\partial \xi} = \frac{h_1}{h_2 h_3}\left\{-\mu \frac{\partial}{\partial \eta}\left(E^2 \psi^{(1)}\right) + \eta_1 \frac{\partial}{\partial \eta}\left(E^4 \psi^{(1)}\right)\right\} \tag{14}$$

$$\frac{\partial p^{(1)}}{\partial \eta} = \frac{h_2}{h_1 h_3}\left\{\mu \frac{\partial}{\partial \xi}\left(E^2 \psi^{(1)}\right) - \eta_1 \frac{\partial}{\partial \xi}\left(E^4 \psi^{(1)}\right)\right\} \tag{15}$$

Eliminating $p^{(1)}$ from (14) and (15), we have

$$\left(E^6 - \frac{\lambda^2}{c^2}E^4\right)\psi^{(1)} = 0\ , \tag{16}$$

where

$$\frac{\lambda^2}{c^2} = \frac{\mu}{\eta_1} \tag{17}$$

Thus the flow variables in the region F_1 are completely determinable from the partial differential equation (16) with the appropriate boundary and regularity conditions. The fluid pressure $p^{(1)}$ can be obtained using equations (14) and (15).

As mentioned earlier, the flow in the porous region F_0 is assumed to be Darcian. In view of this, the equations governing the flow in the region F_0 are given by

$$div\ (\vec{q}^{(0)}) = 0 \tag{18}$$

$$\overline{q}^{(0)} = -\frac{k^{(1)}}{\mu} grad\ \ p^{(0)} \tag{19}$$

which implies that the pressure $p^{(0)}$ is a harmonic function given by the equation

$$\nabla^2 p^{(0)} = 0 \tag{20}$$

4. BOUNDARY CONDITIONS

The determination of the relevant flow field variables $\psi^{(i)}$ and $p^{(i)}$, (i=0, 1) is subjected to the following boundary and regularity conditions.

(i) Continuity of the normal velocity component on the interface:

$$u^{(1)} = u^{(0)} \text{ on } S_1 \tag{21}$$

(ii) Vanishing of the tangential velocity component on the interface:

$$v^{(1)} = 0 \text{ on } S_1 \tag{22}$$

(iii) $$\frac{1}{2} curl\overline{q}^{(1)} = 0 \text{ on } S_1 \tag{23}$$

(iv) No slip condition on S_0 :

$v^{(0)} = 0$ on S_0 (24)

(v) Continuity of pressure on the interface:

$p^{(1)} = p^{(0)}$ on S_1 (25)

In addition to the above boundary conditions, it is natural to have regularity of the flow field variables on the axis of symmetry. Further as the flow is a uniform stream at infinity we have,

$\psi = -\frac{1}{2}Ur^2$ far away from the body. (26)

Other types of boundary conditions are also in vogue. Stokes in [23] mentions also the condition that the couple stresses vanish on the boundary in place of the condition (iii) above. As an initial trial for solving a more difficult problem than in the case of a viscous liquid, we are attempting the problem through the above set of boundary conditions.

5. SOLUTION FOR THE FLOW IN THE REGION F1

Since, we are dealing with a prolate spheroidal coordinate system, we have

$$h_1 = h_2 = c\sqrt{(s^2 - t^2)}\,,\; h_3 = c\sqrt{(s^2 - 1)(1 - t^2)} \tag{27}$$

$$E^2 = \frac{1}{c^2(s^2 - t^2)}\left((s^2 - 1)\frac{\partial^2}{\partial s^2} + (1 - t^2)\frac{\partial}{\partial t^2}\right) \tag{28}$$

$$\nabla^2 = \frac{1}{c^2(s^2 - t^2)}\left((s^2 - 1)\frac{\partial^2}{\partial s^2} + (1 - t^2)\frac{\partial}{\partial t^2} + 2s\frac{\partial}{\partial s} - 2t\frac{\partial}{\partial t}\right) \tag{29}$$

Where

$$s = \cosh\ \xi\ ;\ t = \cos\ \eta \tag{30}$$

We assume that the boundary of the spheroid S_1 is given by $s=s_1$ and that of S_0 by $s=s_0$.

The solution of equation (16) can be obtained by superposing the solutions of the equations

$$E^4\psi = 0 \tag{31}$$

and

$$\left(E^2 - \frac{\lambda^2}{c^2}\right)\psi = 0 \tag{32}$$

in view of the commutativity and linearity of the operators E^4 and $\left(E^2 - \frac{\lambda^2}{c^2}\right)$.

Following Lakshmana Rao and Iyengar [11] and Iyengar, Radhika [25], we see that:

5.1 Solution of Equation (31)

The solution of (31) can be written in the form

$$\psi = \psi_0 + \psi_1 \tag{33}$$

Where

$$\psi_0 = -\frac{1}{2}Uc^2(s^2-1)(1-t^2) \tag{34}$$

and

$$\psi_1 = c^2(s^2-1)(1-t^2)\sum_{n=0}^{\infty} G_{n+1}(s)P'_{n+1}(t) \tag{35}$$

where $P'_{n+1}(t)$ is the derivative of $P_{n+1}(t)$ with respect to t.

where

$$G_{n+1}(s) = B_{n+1}Q'_{n+1}(s) - \frac{P'_{n+1}(s)}{(n+1)(n+2)}\int_{s_1}^{s}(s^2-1)Q'_{n+1}(s)g_{n+1}(s)ds + \frac{Q'_{n+1}(s)}{(n+1)(n+2)}\int_{s_1}^{s}(s^2-1)P'_{n+1}(s)g_{n+1}(s)ds$$

for n=0,1,2… (36)

Here

$$g_{n+1}(s) = c^2\left[\frac{(n+1)(n+2)}{(2n+3)(2n+5)}A_{n+1} - \frac{(n+3)(n+4)}{(2n+5)(2n+7)}A_{n+3}\right]Q'_{n+3}(s) - c^2\left[\frac{(n-1)(n)}{(2n-1)(2n+1)}A_{n-1} - \frac{(n+1)(n+2)}{(2n+1)(2n+3)}A_{n+1}\right]Q'_{n-1}(s) \tag{37}$$

As $g_{n+1}(s)$ involves one set $\{A_{n+1}\}$ of arbitrary constants, the functions $G_{n+1}(s)$ involve two sets of arbitrary constants $\{A_{n+1}\}$ and $\{B_{n+1}\}$. Using this in equation (35), we get ψ_1.

5.2 Solution of Equation (32)

To solve the equation (32) (viz.) $\left(E^2 - \frac{\lambda^2}{c^2}\right)\psi = 0$, we use the method of separation of variables* and take the solution in the form

$$\psi = c\sqrt{(s^2-1)(1-t^2)}R(s)S(t) \tag{38}$$

Substituting (38) in the equation (32), we notice that *R(s)* and *S(t)* respectively satisfy the differential equations

$$(s^2-1)R''(s) + 2s\,R'(s) - \left(\Lambda + \lambda^2 s^2 + \frac{1}{s^2-1}\right)R(s) = 0 \tag{39}$$

and

$$\left(1-t^2\right)S''(t)-2t\,S'(t)+\left(\Lambda+\lambda^2t^2-\frac{1}{1-t^2}\right)S(t)=0 \tag{40}$$

where Λ is a separation constant [25]. These are spheroidal wave differential equations of radial and angular type respectively. To ensure regularity of solution at infinity and in the flow region, we have to choose the solutions of equations (39) and (40) in the form

$$R_{1n}^{(3)}(i\lambda,s)=\left[i^{n+2}\sum_{r=0,1}^{\infty}{}'(r+1)(r+2)d_r^{1n}(i\lambda)\right]^{-1} \left(\frac{s^2-1}{s^3}\right)^{1/2}\left(\frac{2}{\pi\lambda}\right)^{1/2}\sum_{r=0,1}^{\infty}{}'(r+1)(r+2)d_r^{1n}(i\lambda)K_{r+3/2}(\lambda s) \tag{41}$$

and

$$S_{1n}^{(1)}(i\lambda,t)=\sum_{r=0,1}^{\infty}{}'d_r^{1n}(i\lambda)P_{r+1}^{(1)}(t) \tag{42}$$

where

$$P_{r+1}^{(1)}(t)=\sqrt{1-t^2}\,\frac{d}{dt}P_{r+1}(t) \tag{43}$$

denotes the associated Legendre function of the first kind. In view of this, the solution of equation (32) is given by

$$\psi_2=c\sqrt{(s^2-1)(1-t^2)}\sum_{n=1}^{\infty}C_nR_{1n}^{(3)}(i\lambda,s)S_{1n}^{(1)}(i\lambda,t) \tag{44}$$

where C_n's are constants.

Hence, the stream function for the region F$_1$ is given by

$$\psi^{(1)}(s,t)=-\frac{1}{2}Uc^2(s^2-1)(1-t^2)+ c^2(s^2-1)(1-t^2)\sum_{n=0}^{\infty}G_{n+1}(s)P'_{n+1}(t)+ c\sqrt{(s^2-1)(1-t^2)}\sum_{n=1}^{\infty}C_nR_{1n}^{(3)}(i\lambda,s)S_{1n}^{(1)}(i\lambda,t) \tag{45}$$

We can see that

$$E^2\psi^{(1)}=c^2(s^2-1)(1-t^2)\sum_{n=0}^{\infty}A_{n+1}Q'_{n+1}(s)P'_{n+1}(t)+ \frac{\lambda^2}{c}\sqrt{(s^2-1)(1-t^2)}\sum_{n=1}^{\infty}C_nR_{1n}^{(3)}(i\lambda,s)S_{1n}^{(1)}(i\lambda,t) \tag{46}$$

and

$$E^4\psi^{(1)} = \frac{\lambda^4}{c^3}\sqrt{(s^2-1)(1-t^2)}\sum_{n=1}^{\infty} C_n R_{1n}^{(3)}(i\lambda,s)S_{1n}^{(1)}(i\lambda,t) \tag{47}$$

and these are recorded here for future use.

5.3 Pressure Distribution in Ғ$_1$

The equations (14) and (15), using equation (30) lead to

$$\frac{\partial p^{(1)}}{\partial s} = \frac{\mu}{2c(s^2-1)}\frac{\partial}{\partial t}\left(E^2\psi^{(1)}\right) - \frac{\eta_1}{2c(s^2-1)}\frac{\partial}{\partial t}\left(E^4\psi^{(1)}\right)$$

and

$$\frac{\partial p^{(1)}}{\partial t} = -\frac{\mu}{2c(1-t^2)}\frac{\partial}{\partial s}\left(E^2\psi^{(1)}\right) + \frac{\eta_1}{2c(1-t^2)}\frac{\partial}{\partial s}\left(E^4\psi^{(1)}\right) \tag{48}$$

Using the expressions given in equations (45) and (46**)**, on integration we get

$$p^{(1)}(s,t) = -\mu c\sum_{n=0}^{\infty} A_{n+1}Q_{n+1}(s)(n+1)(n+2)P_{n+1}(t) \tag{49}$$

Thus $\psi^{(1)}(s,t)$ and $p^{(1)}(s,t)$ given in equations (44) and (49) give respectively the stream function and pressure distribution for the region Ғ$_1$. It can be seen that these involve three sets of constants i.e $\{A_n\}, \{B_n\}, \{C_n\}$.

6. SOLUTION FOR THE FLOW IN THE REGION Ғ$_0$

We have seen earlier that the flow in the porous region Ғ$_0$ is governed by the equations (18) and (19) which lead to the equation (20). The equation (20) implies that the pressure distribution $p^{(0)}(s,t)$ in Ғ$_0$ is harmonic and hence it is given by

$$p^{(0)}(s,t) = \sum_{n=0}^{\infty}\left(\alpha_n P_n(s) + \beta_n Q_n(s)\right)P_n(t) \tag{50}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ constitute two sets of arbitrary constants to be determined. The velocity components $u^{(0)}(s,t)$ and $v^{(0)}(s,t)$ can be determined from equations (19) and (50).

In view of the continuity equation in the region Ғ$_0$, we introduce the stream function $\psi^{(0)}$ through

$$h_2h_3u^{(0)} = -\frac{\partial\psi^{(0)}}{\partial\eta}\ ;\ h_1h_3v^{(0)} = \frac{\partial\psi^{(0)}}{\partial\xi} \tag{51}$$

as in equation (11).Using equations(51) and (19), the stream function $\psi^{(0)}$ takes the form

$$\psi^{(0)}(s,t) = -k^{(1)}c(s^2-1) \sum_{n=0}^{\infty}\left(\alpha_{2n+1}P'_{2n+1}(s)+\beta_{2n+1}Q'_{2n+1}(s)\right)\int_{-1}^{t}P_{2n+1}(t)dt \tag{52}$$

Thus, in all, we have five sets of unknown constants $\{A_n\}$, $\{B_n\}$, $\{C_n\}$, $\{\alpha_n\}$ and $\{\beta_n\}$ and these can be determined by using the boundary conditions given by the equations (21)-(25).

7. VELOCITY AND MICROROTATION COMPONENTS IN THE REGIONS Ɨ$_0$, Ɨ$_1$

The expressions for the velocity components $u^{(1)}(s,t)$ and $v^{(1)}(s,t)$ are

$$u^{(1)}(s,t) = \frac{1}{c^2\sqrt{(s^2-t^2)(s^2-1)}}\frac{\partial\psi^{(1)}}{\partial t}$$

$$v^{(1)}(s,t) = \frac{1}{c^2\sqrt{(s^2-t^2)(1-t^2)}}\frac{\partial\psi^{(1)}}{\partial s} \tag{53}$$

Further

$$u^{(0)}(s,t) = -\frac{k^{(1)}\sqrt{s^2-1}}{c\sqrt{(s^2-t^2)}}\frac{\partial p^{(0)}}{\partial s}$$

$$v^{(0)}(s,t) = \frac{k^{(1)}\sqrt{1-t^2}}{c\sqrt{(s^2-t^2)}}\frac{\partial p^{(0)}}{\partial t} \tag{54}$$

These can be obtained by using the expressions for $\psi^{(1)}$ given in equations (44) and $p^{(0)}$ given in equation (50). Thus the expressions for the velocity components $u^{(1)}$, $v^{(1)}$; $u^{(0)}$, $v^{(0)}$; can all be written explicitly. Using these expressions and those of $p^{(0)}$ and $p^{(1)}$ in the boundary conditions given by equations (21) - (25) we can write the equations that lead to the determination of the arbitrary constants.

8. DETERMINATION OF ARBITRARY CONSTANTS

Here again, the procedure is similar to that in [11]. In view of the continuity of the normal velocity components on the interface s=s$_1$ given by equation (21), we have

$$Uc^2\left(s_1^2-1\right)-c^2\left(s_1^{\,2}-1\right)\sum_{n=0}^{\infty}G_{n+1}(s_1)(n+1)(n+2)P_{n+1}(t) - c\sqrt{s_1^{\,2}-1}\sum_{n=1}^{\infty}C_nR_{1n}^{(3)}(i\lambda,s_1)\frac{d}{dt}\left(\sqrt{1-t^2}\,S_{1n}^{(1)}(i\lambda,t)\right)= -\frac{k^{(1)}}{\mu}c\left(s_1^2-1\right)\sum_{n=0}^{\infty}\left(\alpha_{n+1}P'_{n+1}(s_1)+\beta_{n+1}Q'_{n+1}(s_1)\right)P_{n+1}(t) \tag{55}$$

As the tangential velocity component is to vanish on the boundary s = s$_1$, the equation (22) leads to

$$-Uc^2 s_1(1-t^2)P_1'(t)+c^2\sum_{n=0}^{\infty}\frac{d}{ds}\left((s^2-1)G_{n+1}(s)\right)_{s=s_1}(1-t^2)P_{n+1}'(t)+ \\ c\sum_{n=1}^{\infty}C_n\frac{d}{ds}\left[\sqrt{s^2-1}R_{1n}^{(3)}(i\lambda,s)\right]_{s=s_1}\sum_{r=0,1}^{\infty}{}' d_r^{1n}(i\lambda)(1-t^2)P_{r+1}'(t)=0 \quad (56)$$

The condition (23) yields

$$c\sqrt{(s_1{}^2-1)}\sum_{n=0}^{\infty}A_{n+1}Q_{n+1}'(s_1)\sqrt{1-t^2}P_{n+1}'(t)+\frac{\lambda^2}{c^2}\sum_{n=1}^{\infty}C_nR_{1n}^{(3)}(i\lambda,s_1)\sum_{r=0,1}^{\infty}{}' d_r^{1n}(i\lambda)\sqrt{1-t^2}P_{r+1}'(t)=0 \quad (57)$$

The no slip condition on S_0 given by (24) leads to

$$-\frac{k^{(1)}\sqrt{(1-t^2)}}{c\sqrt{s^2-t^2}}\sum_{n=0}^{\infty}\left(\alpha_{n+1}P_{n+1}(s_0)+\beta_{n+1}Q_{n+1}(s_0)\right)P_{n+1}'(t)=0 \quad (58)$$

The continuity of pressure on the interface $s=s_1$ given by equation (25) yields

$$-\mu c\sum_{n=0}^{\infty}A_{n+1}Q_{n+1}(s_1)(n+1)(n+2)P_{n+1}(t)= \\ \sum_{n=0}^{\infty}\left(\alpha_{n+1}P_{n+1}(s_1)+\beta_{n+1}Q_{n+1}(s_1)\right)P_{n+1}(t) \quad (59)$$

Using the orthogonality property of Legendre functions and the associated Legendre functions, the equations (55)-(59) give rise to the following equations adopting some simple algebraic manipulation:

$$Uc^2\left(s_1^2-1\right)\delta_{n0}-c^2\left(s_1{}^2-1\right)B_{n+1}Q_{n+1}'(s_1)(n+1)(n+2)- \\ c(n+1)(n+2)\sum_{m=1}^{\infty}C_m\sqrt{s_1{}^2-1}R_{1m}^{(3)}(i\lambda,s_1)d_n^{1m}(i\lambda)= \\ -\frac{k^{(1)}}{\mu}c\left(s_1^2-1\right)\left(\alpha_{n+1}P'_{n+1}(s_1)+\beta_{n+1}Q'_{n+1}(s_1)\right) \quad (60)$$

$$-Uc^2 s_1\delta_{0n}+c^2B_{n+1}(n+1)(n+2)Q_{n+1}(s_1)+ \\ c\sum_{m=1}^{\infty}C_m\frac{d}{ds}\left[\sqrt{s^2-1}R_{1n}^{(3)}(i\lambda,s)\right]_{ons=s_1}d_n^{1m}(i\lambda)=0 \quad (61)$$

$$c\sqrt{(s_1{}^2-1)}A_{n+1}Q_{n+1}'(s_1)+\frac{\lambda^2}{c^2}\sum_{m=1}^{\infty}C_mR_{1m}^{(3)}(i\lambda,s_1)d_n^{1m}(i\lambda)=0 \quad (62)$$

$$-\mu\, c\, A_{n+1}Q_{n+1}(s_1)(n+1)(n+2)=\alpha_{n+1}P_{n+1}(s_1)+\beta_{n+1}Q_{n+1}(s_1) \quad (63)$$

$$\alpha_{n+1}P_{n+1}(s_0)+\beta_{n+1}Q_{n+1}(s_0)=0 \quad (64)$$

From equations (60) and (61), the coefficient B_{n+1} can be eliminated and using (62)- (64), we get a non- homogeneous linear system of algebraic equations for the determination of constants $\{C_n\}$. This system is seen to be

$$\sum_{m=1}^{\infty} D_{nm} C_m = -U c \delta_{0n} \text{ , n=0,1,2…} \tag{65}$$

Where

$$D_{nm} = d_{2n}^{1m}(i\lambda) \begin{bmatrix} \left\{ \sqrt{s_1^{\,2}-1}\,\dfrac{d}{ds} R_{1m}^{(3)}(i\lambda, s_1) + \dfrac{s_1}{\sqrt{s_1^{\,2}-1}} R_{1m}^{(3)}(i\lambda, s_1) \right\} (s_1^{\,2}-1) Q'_{2n+1}(s_1) \\ -2(n+1)(2n+1)\sqrt{s_1^{\,2}-1}\,Q_{2n+1}(s_1) R_{1m}^{(3)}(i\lambda, s_1) \\ -(n+1)(2n+1)\dfrac{\lambda^2}{c^2} 2k^{(1)}(s_1^{\,2}-1)\dfrac{Q_{2n+1}(s_1)}{Q'_{2n+1}(s_1)}\dfrac{Q_{2n+1}(s_1)}{\sqrt{s_1^{\,2}-1}} R_{1m}^{(3)}(i\lambda, s_1) \\ \dfrac{Q_{2n+1}(s_0)P'_{2n+1}(s_1) - P_{2n+1}(s_0)Q'_{2n+1}(s_1)}{P_{2n+1}(s_0)Q_{2n+1}(s_1) - P_{2n+1}(s_1)Q_{2n+1}(s_0)} \end{bmatrix} \tag{66}$$

The above linear system splits into two complementary sub systems where n is even and n is odd. The subsystem when n is odd reduces to the homogeneous set of equations

$$\sum_{m=1}^{\infty} D_{2n+1,2m} C_{2m} = 0 \tag{67}$$

and we therefore have $C_2 = C_4 = C_6 \ldots = 0$. Hence A_n, B_n are all zero when n is even. The explicit analytical determination of the odd suffixed constants $\{C_n\}$ is not possible. In view of this, we propose to determine them numerically. Here we truncate the system (67) to fifth order and numerically evaluate the coefficients C_1, C_3, C_5, C_7 and C_9. This is the maximum extent to which the order of truncation can be extended since the coefficients of spheroidal wave functions needed for a higher order truncation are not explicitly available in the standard literature [26].

After determining these, it is possible to evaluate numerically the other constants. The details of the manipulations are omitted in view of the lengthiness of the expressions and the final system only is reported here.

9. DETERMINATION OF DRAG

To evaluate the drag on the body, we need the stress components and the couple stress components. The stress tensor is given by equation (1) and we need to evaluate the rate of strain components e_{ij} and the spin component ω_ϕ

The velocity vector $\vec{q}$ can be written in the form

$$\vec{q} = u\,\vec{e}_\xi + v\,\vec{e}_\eta \tag{68}$$

Where

$$u = \frac{1}{c^2\sqrt{(s^2-t^2)(s^2-1)}}\frac{\partial \psi}{\partial t}$$

$$v = \frac{1}{c^2\sqrt{(s^2-t^2)(1-t^2)}}\frac{\partial\psi}{\partial s} \tag{69}$$

The rate of strain components are given by

$$e_{\xi\xi} = \frac{1}{c^3(s^2-t^2)}\left(\psi_{st} + \frac{t}{s^2-t^2}\psi_s - \frac{s(2s^2-1-t^2)}{(s^2-t^2)(s^2-1)}\psi_t\right)$$

$$e_{\xi\eta} = e_{n\xi} = \frac{(s^2-1)\psi_{ss} - (1-t^2)\psi_{tt}}{2c^3(s^2-t^2)\sqrt{(s^2-1)(1-t^2)}} - \frac{s\sqrt{s^2-1}}{c^3(s^2-t^2)^2\sqrt{1-t^2}}\psi_s - \frac{t\sqrt{1-t^2}}{c^3(s^2-t^2)^2\sqrt{s^2-1}}\psi_t \tag{70}$$

$$e_{\eta\eta} = \frac{1}{c^3(s^2-t^2)}\left(-\psi_{st} + \frac{s}{s^2-t^2}\psi_t + \frac{t(2t^2-1-s^2)}{(s^2-t^2)(1-t^2)}\psi_s\right)$$

$$e_{\phi\phi} = \frac{1}{c^3(s^2-t^2)}\left(\frac{s}{s^2-1}\psi_t + \frac{t}{(1-t^2)}\psi_{st}\right)$$

$$e_{\xi\phi} = e_{\phi\xi} = e_{\eta\phi} = e_{\phi\eta} = 0$$

The spin = $\frac{1}{2}curl\,\vec{q}$ has only one non zero component ω_ϕ in the direction of the vector $\vec{e}_\phi$ and this is given by

$$\omega_\phi = \frac{1}{2c\sqrt{(s^2-1)(1-t^2)}}E^2\psi \tag{71}$$

The surface stress t_{ij} for the couple stress fluid is given by equation (5) and we find that the only non-vanishing components of t_{ij} are $t_{\xi\xi}$, $t_{\eta\eta}$, $t_{\phi\phi}$, $t_{\xi\eta}$ and $t_{\eta\xi}$. These are given by

$$t_{\xi\xi} = -p + 2\mu e_{\xi\xi}$$

$$t_{\eta\eta} = -p + 2\mu e_{\eta\eta} \tag{72}$$

$$t_{\phi\phi} = -p + 2\mu e_{\phi\phi}$$

$$t_{\xi\eta} = 2\mu e_{\xi\eta} - \frac{\eta_1}{h_3}E^4\psi^{(1)}$$

$$t_{\eta\xi} = 2\mu e_{\eta\xi} + \frac{\eta_1}{h_3} E^4 \psi^{(1)}$$

The stress vector $\vec{t}$ on the boundary of the body is given by

$$\vec{t} = t_{\xi\xi}\, \vec{\mathrm{e}}_\xi + t_{\xi\eta}\, \vec{\mathrm{e}}_\eta \tag{73}$$

We find that

$$\left(t_{\xi\xi}\right)_{s=s_1} = -p^{(1)}(s_1,t) + \frac{2k^{(1)} s_1 (2{s_1}^2 - 1 - t^2)}{c^2 ({s_1}^2 - t^2)^2} \sum_{n=0}^{\infty} \left(\alpha_{n+1} P'_{n+1}(s_1) + \beta_{n+1} Q'_{n+1}(s)\right) P_{n+1}(t) \tag{74}$$

And

$$\begin{aligned}\left(t_{\xi\eta}\right)_{\mathrm{s}=\mathrm{s}_1} = {} & \mu c \sqrt{(\mathrm{s}_1{}^2 - 1)(1 - t^2)} \sum_{n=0}^{\infty} A_{n+1} Q'_{n+1}(s_1) P'_{n+1}(t) + \\ & 2\mu \frac{\lambda^2}{c^2} \sum_{n=0}^{\infty} C_n R_{1n}^{(3)}(i\lambda, s_1) S_{1n}^{(1)}(i\lambda, t) + \\ & \frac{2k^{(1)}}{c^2 ({s_1}^2 - t^2)} \sqrt{(\mathrm{s}_1{}^2 - 1)(1 - t^2)} \sum_{n=0}^{\infty} (\alpha_{n+1} P'_{n+1}(s_1) + \beta_{n+1} Q'_{n+1}(s)) P'_{n+1}(t) + \\ & \frac{2tk^{(1)}}{c^2 ({s_1}^2 - t^2)^2} \sqrt{(\mathrm{s}_1{}^2 - 1)(1 - t^2)} \sum_{n=0}^{\infty} (\alpha_{n+1} P'_{n+1}(s_1) + \beta_{n+1} Q'_{n+1}(s)) P_{n+1}(t) - \\ & \eta_1 \frac{\lambda^4}{c^4} \sum_{n=1}^{\infty} C_n R_{1n}^{(3)}(i\lambda, s) S_{1n}^{(1)}(i\lambda, t)\end{aligned} \tag{75}$$

The stress vector has the component

$$\left(stress\right)_{axial} = \frac{1}{\sqrt{{s_1}^2 - t^2}} \left(t\sqrt{s^2 - 1}\ \mathrm{t}_{\xi\xi} - s\sqrt{1 - t^2}\ \mathrm{t}_{\xi\eta}\right)_{s=s_1} \tag{76}$$

in the direction of the axis of symmetry and

$$\left(stress\right)_{radial} = \frac{1}{\sqrt{{s_1}^2 - t^2}} \left(s\sqrt{1 - t^2}\ \mathrm{t}_{\xi\xi} + t\sqrt{s^2 - 1}\ \mathrm{t}_{\xi\eta}\right)_{s=s_1} \tag{77}$$

in the radial direction of the meridian plane. The resultants of these two vector components over the entire surface of the body are obtained by integration and it is seen that the radial component integrates to zero. Thus the resultant of the stress vector on the body is the force in the direction of the axis of symmetry and this gives the drag on the body. The drag D can be written in the form

$$D = 2\pi\, c^2 \sqrt{s_1{}^2 - 1}\int_{-1}^{1}\left(t\sqrt{s^2-1}\ \mathrm{t}_{\xi\xi} - s\sqrt{1-t^2}\ \mathrm{t}_{\xi\eta}\right)_{s=s_1} dt \tag{78}$$

and this simplifies to

$$2\pi\, c^2\sqrt{s_1{}^2-1}\left(\begin{array}{l} \displaystyle\int_{-1}^{1} t\sqrt{s_1{}^2-1}\; p^{(1)}(s_1,t)dt + \\ \qquad \displaystyle\frac{4k^{(1)}s_1(s_1{}^2-1)^{3/2}}{c^2}\sum_{n=0}^{\infty}\left(\alpha_{n+1}P'_{n+1}(s)+\beta_{n+1}Q'_{n+1}(s)\right)\int_{-1}^{1}\frac{tP_{n+1}(t)}{(s_1{}^2-t^2)^2}dt - \\ \qquad \displaystyle\frac{2\,\mathrm{k}^{(1)}}{c^2}s_1\sqrt{s_1{}^2-1}\sum_{n=0}^{\infty}\left(\alpha_{n+1}P'_{n+1}(s)+\beta_{n+1}Q'_{n+1}(s)\right)\int_{-1}^{1}\frac{(1-t^2)P'_{n+1}(t)}{(s_1{}^2-t^2)}dt - \\ \qquad \displaystyle\eta_1\left(\frac{\lambda^2}{c^2}\right)^2 s_1\sum_{n=0}^{\infty}C_n R_{1n}^{(3)}(i\lambda,s_1)\int_{-1}^{1}\sqrt{(1-t^2)}S_{1n}^{(1)}(i\lambda,t)dt \end{array}\right) \tag{79}$$

Using the relations

$$\int_{-1}^{1}\frac{(1-t^2)P'_n(t)}{(s_1{}^2-t^2)}dt = -\frac{2}{s_1}(s_1{}^2-1)Q'_n(s_1) \tag{80}$$

and

$$\int_{-1}^{1}\frac{tP_n(t)}{(s_1{}^2-t^2)^2}dt = -\frac{1}{s_1}Q'_n(s_1)\,, \tag{81}$$

drawn from " The Theory of Spherical and Ellipsoidal Harmonics" due to Hobson [27], the drag simplifies to

$$D = 2\pi\,\mathrm{c}\sqrt{s_1{}^2-1}\left(\begin{array}{l} \displaystyle\frac{4}{3}\mu c A_1\sqrt{s_1{}^2-1}\left(Q_1(s_1)-s_1Q'_1(s_1)\right) \\ \qquad\qquad \displaystyle -\frac{4}{3}\mu\frac{\lambda^2}{c^2}s_1\sum_{n=0}^{\infty}{}' C_n R_{1n}^{(3)}(i\lambda,s_1)d_0^{1n}(i\lambda) \end{array}\right) \tag{82}$$

Using the equation (65) we may eliminate the series involving the constants C_n in the above expression for the drag and after further simplification we see that the drag due to the surface stress is given by the simple formula

$$\mathrm{D} = \frac{8}{3}\mu\pi\, c^3 A_1 \tag{83}$$

Introducing the non dimensionalization scheme given by

$$A_{n+1} = \frac{U}{c^2}\widetilde{A}_{n+1}\ ;\ C_n = Uc\widetilde{C}_n\ ;\ D = 8\pi\mu Uc\widetilde{D} \tag{84}$$

It is seen, after dropping the tildes, the non dimensional drag D is given by

$$\mathrm{D} = \frac{1}{3}A_1 \tag{85}$$

Where

$$A_1 = -\frac{\mu}{\eta_1}\frac{\displaystyle\sum_{m=1}^{\infty}{}' C_m R_{1m}^{(3)}(i\lambda,s_1)d_0^{1m}(i\lambda)}{\sqrt{s_1{}^2-1}Q'_1(s_1)} \tag{86}$$

This depends upon the eccentricity of the spheroid, the material constant λ, and the non dimensional permeability parameter defined through $kp = \frac{k^{(1)}}{c^2}$.

10. NUMERICAL DISCUSSION

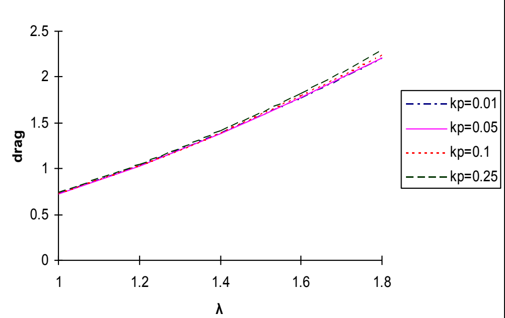
The drag on the spheroidal shell is numerically evaluated for several parameter values by computing the values of the constants $\{C_{2m+1}\}$ by truncating the system in the equation (67) as commented earlier. The values of these constants are plugged into equation (83) and hence the drag D given in equation (86) is evaluated for diverse values of λ and kp. The variation of the drag is displayed through Figs. (1) to (4).

For each value of the permeability parameter kp, the drag is increasing as λ increases. An increase in λ implies a decrease in the couple stress viscosity η. Hence, we notice that as resistance to rotation decreases, the body experiences a greater drag. For a fixed λ, for an increase in kp, the drag is seen to be slightly increasing (see Fig. 1.) and the increase is not significant.

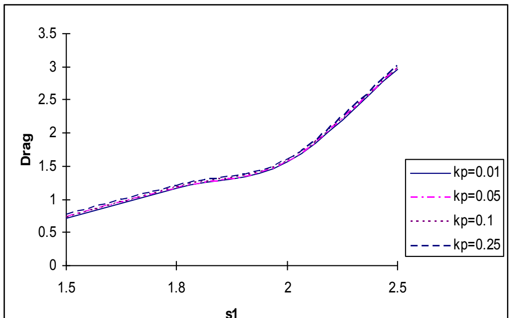
An increase in s_1 indicates an increase in the size of the outer spheroid. The Fig. 2 shows that as the size of the outer spheroid increases, for a fixed λ, when the size of the inner spheroid is fixed, the drag is increasing. Here again, the permeability kp has no significant influence on the drag.

In Fig. 3, we plotted the variation of drag for fixed values of kp and s_0, with respect to varying s_1 and diverse values of λ. Here also we note that as the size of the outer spheroid increases, the drag increases. Also as the couple stress parameter λ increases, the drag is significantly influenced. Fig. 4 also indicates this aspect.

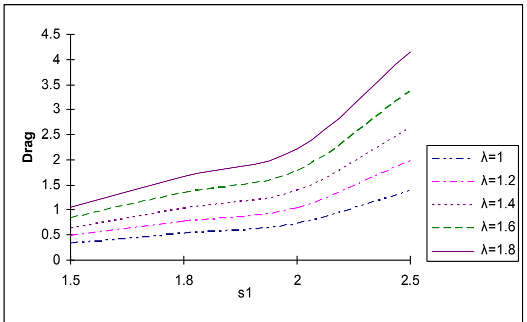
The Figs. 5 to 10 indicate the streamline pattern for diverse values of the couple stress parameter λ and permeability parameter kp. The evaluation of the stream function needs the evaluation of the coefficients d_n^{mn}, $Q_n(s)$, $Q_n'(s)$, $P_n(t)$, $P_n'(t)$, $R_{1n}^{(3)}(i\lambda, s)$, $S_{1n}^{(1)}(i\lambda, t)$ and their derivatives at diverse values of (s,t). For these, program is developed in C and the stream function is evaluated.



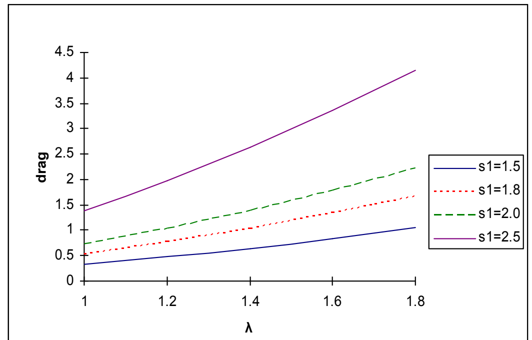
Fig(1) : Variation of drag with respect to λ for different values of the permeability parameter kp when s0=1.2 and s1=2.0



fig(2): Variation of drag with s1 for different values of the permeability parameter kp when λ=1.5 and s0=1.2



fig(3): Variation of drag with respect to s1 for different values of λ at kp=0.1 and s0=1.2



fig(4) : Variation of drag with respect to λ for different s1 when s0=1.2 and the permeability parameter kp=0.1

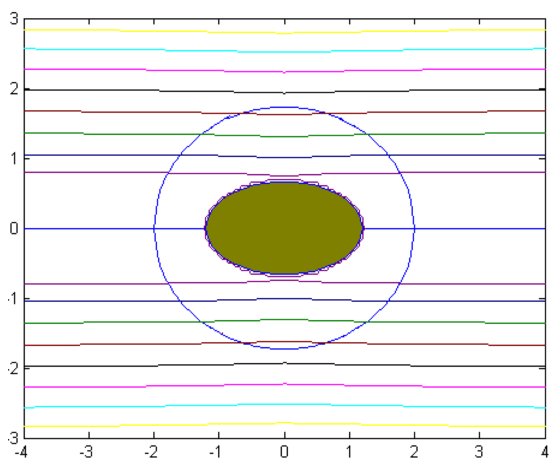


Fig. 5. Streamline pattern for λ =0.5, kp=0.01

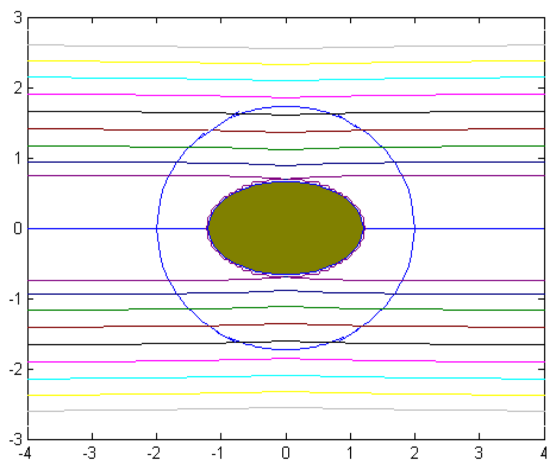


Fig. 6. Streamline pattern for λ =0.5, kp=0.5

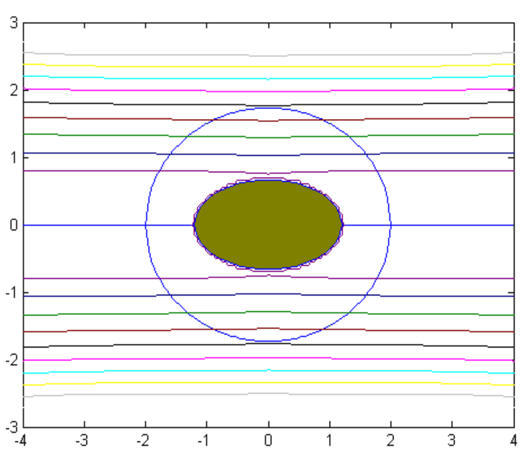


Fig. 7. Streamline pattern for λ =1.5, kp=0.01

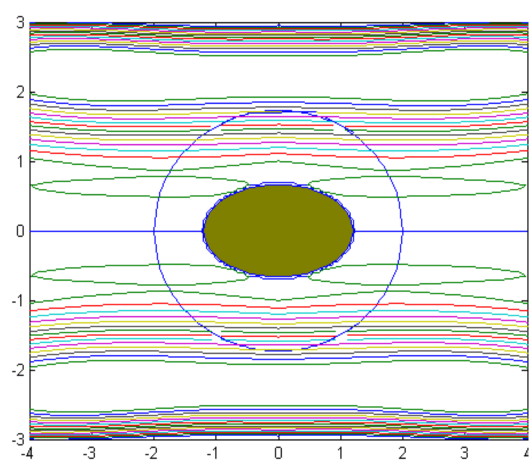


Fig. 8. Streamline pattern for λ =2.0

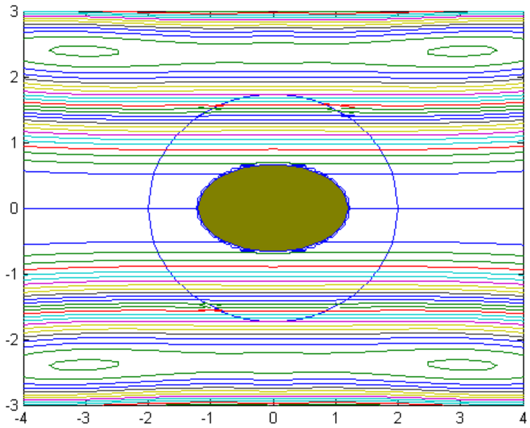


Fig. 9. Streamline pattern for λ =3.0, kp=0.25

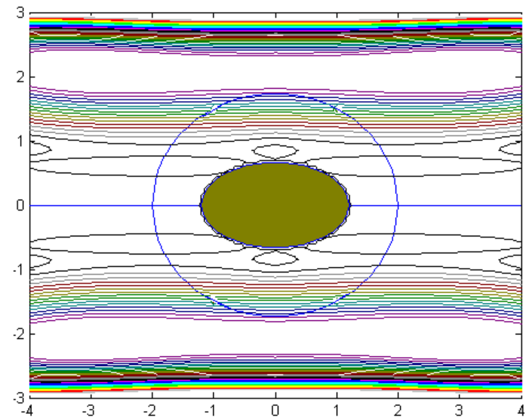


Fig. 10. Streamline pattern for λ =2.5, kp=0.5

For λ = 0.5 or 1.5 and kp = 0.01 or 0.05 there is not much of disturbance in the stream lines.

However as λ as well as kp are simultaneously increasing, the flow pattern is disturbed and dividing streamlines are appearing in the flow field. A suitable experimental set up is needed to verify this observation.

11. CONCLUSION

In this paper, the authors investigated the slow steady flow of an incompressible couple stress fluid past a porous spheroidal shell containing a rigid confocal spheroidal core and evaluated the drag experienced by the body. It has been found that the drag experienced by the body is increased with the increase in the size of the outer spheroid and also by an increase in the permeability parameter. The couple stress parameter also has a significant effect on the drag. Stream lines depicting the flow are also plotted. For greater values of the couple stress parameter and the permeability parameter, a dividing streamline pattern has been obtained.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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